Bubble universes, and signals from before

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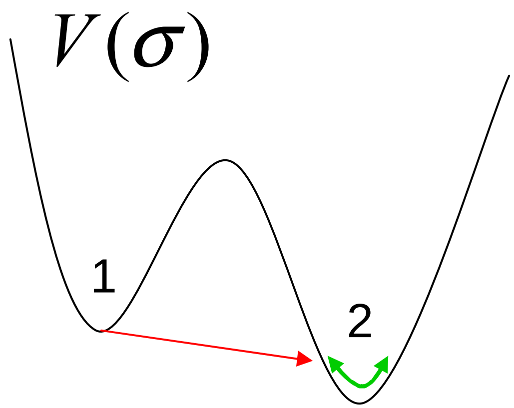
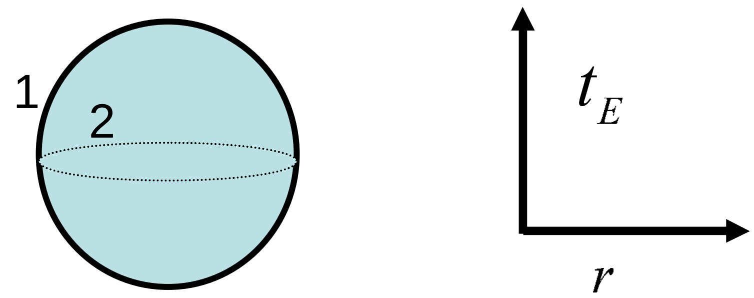
(Palau de la Musica)

False vacuum decay



$$\mathrm{Im}\,(E_1) = A\ e^{-B}$$

$$B = I_E[inst] - I_E[1]$$



$$\boxed{\sigma = \sigma_0\,(r^2 + t_E^2)}$$ instanton

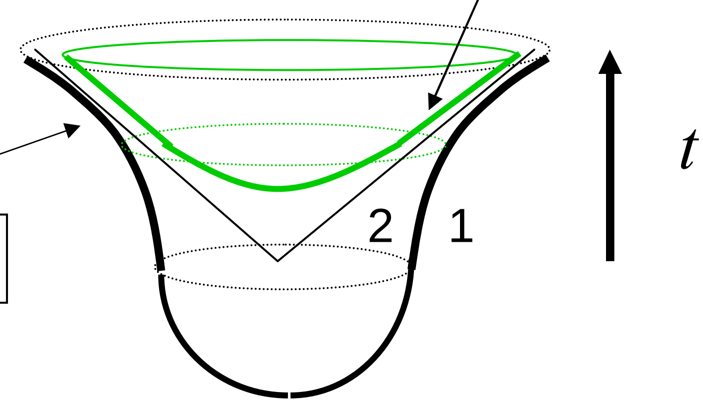
LORENTZIAN EVOLUTION AFTER TUNNELING

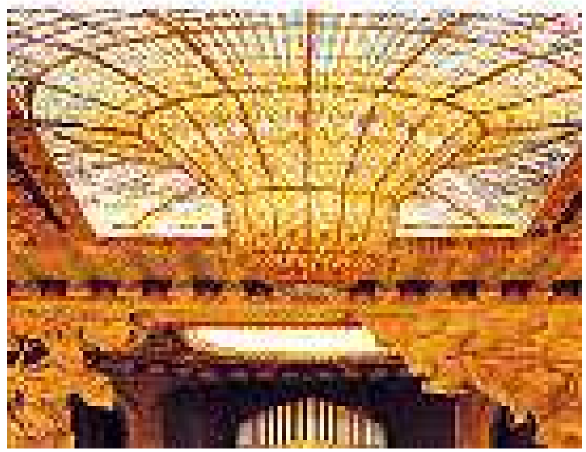
$$t_E \rightarrow i\ t$$

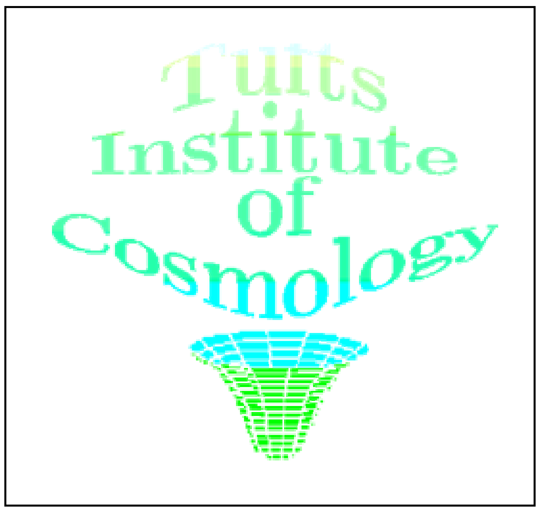
$O(3\,,1)$ symmetric

$\boxed{r^2 - t^2 = const.}$

constant energy density surface

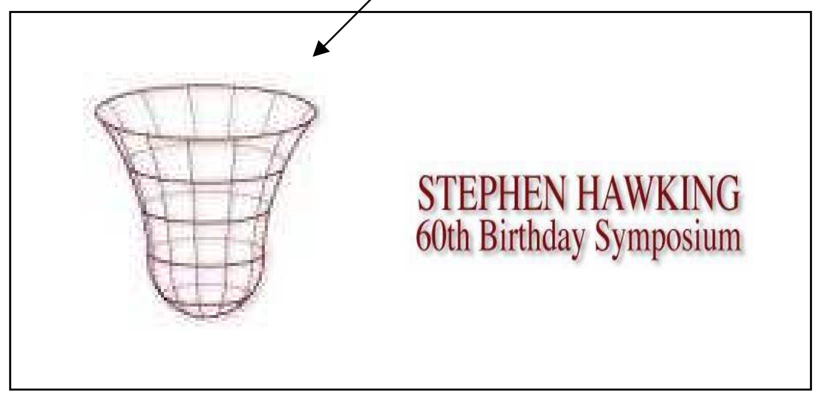




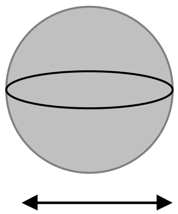


Tunneling from nothing

Inflating (dS)



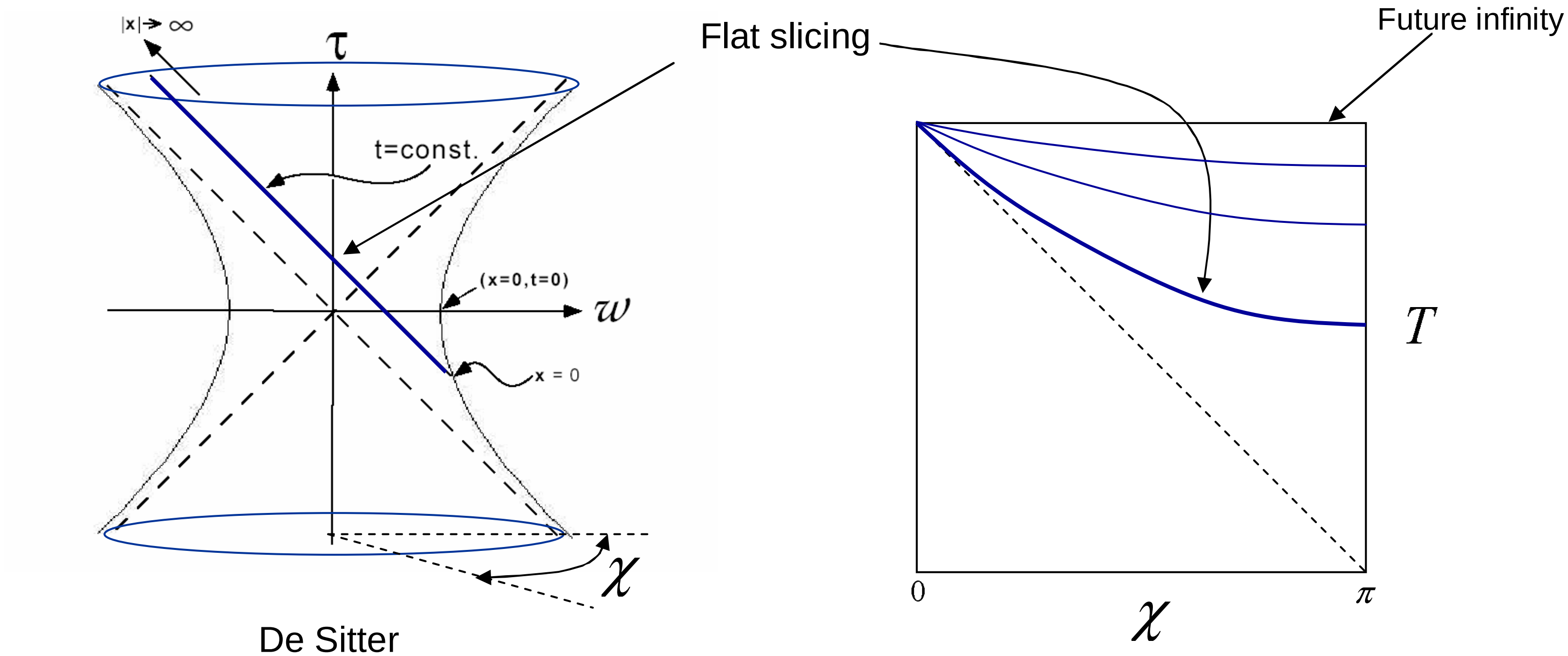
No boundary proposal



$H^{-1} \sim \Lambda^{-1/2}$

4 sphere

$$ds^2 = -dt^2 + e^{2Ht}\ (d\vec{x})^2$$

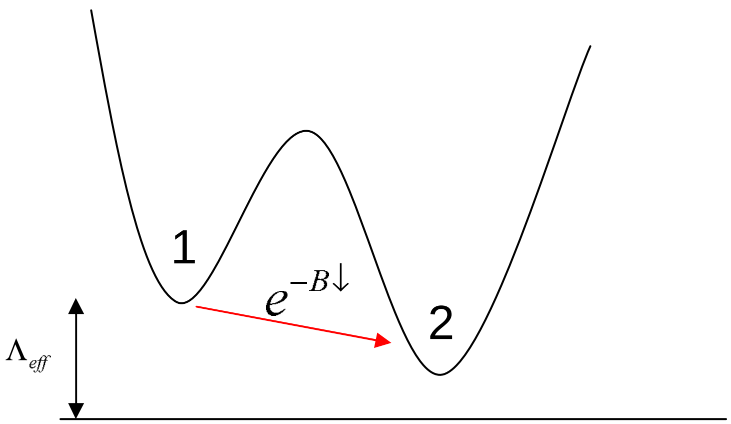


De Sitter

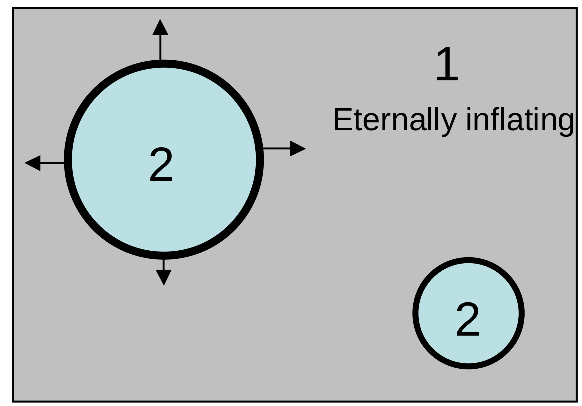
(pseudo-sphere)

Conformal diagram

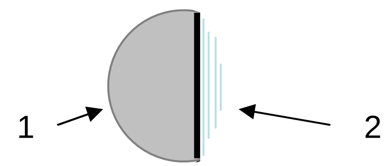
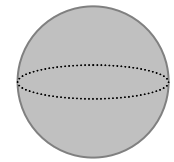
dS “vacua” generically lead to eternal inflation



For $\Gamma \sim A\, e^{-B} \ll H^4$
bubbles of the new phase
do not percolate

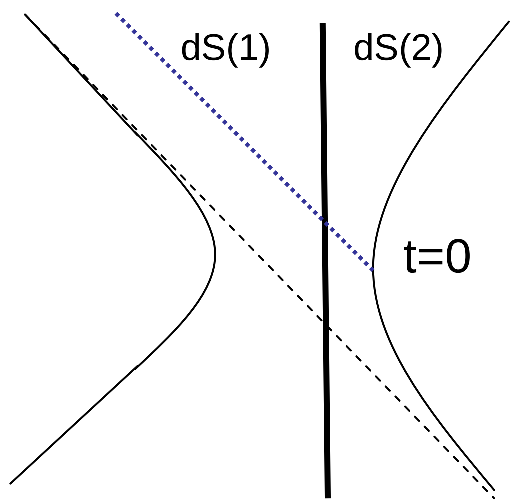


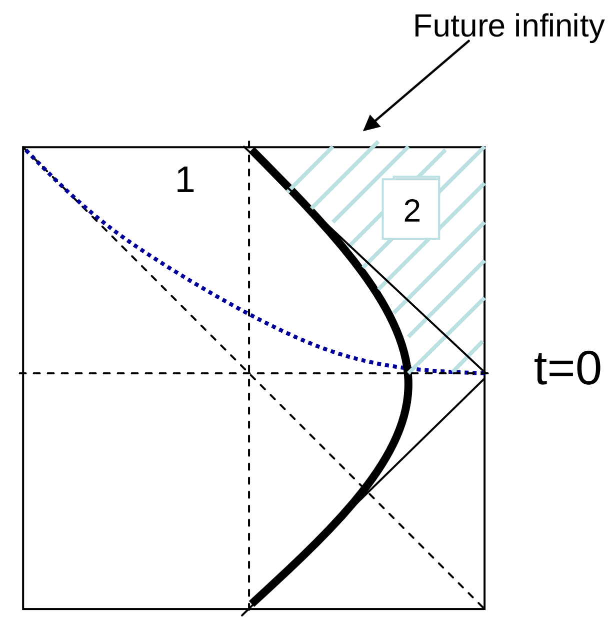
Inflating volume in false vacuum $V_1(t) > C\, e^{(3 - H^{-4}\Gamma)\, Ht}$

$B^{\downarrow} =$  $-$ 

Coleman - de Luccia instanton (CdL)

dS (1)



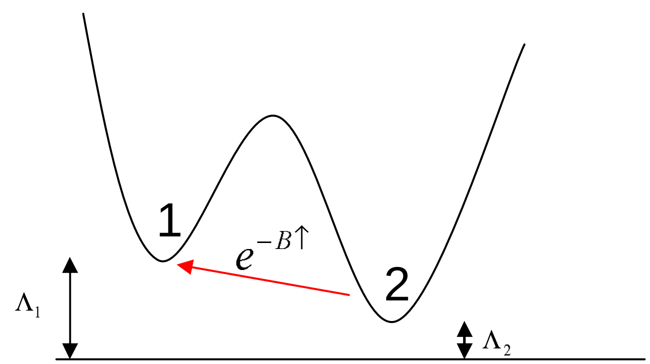


Conformal diagram

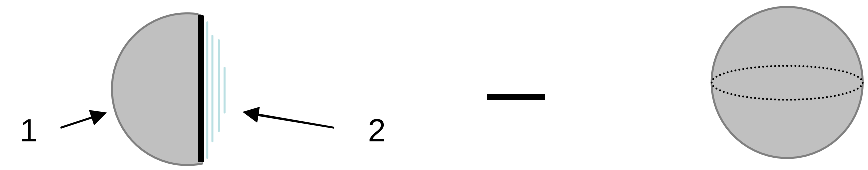
Tunneling uphill
is also possible

$$\Gamma^{\uparrow}/\Gamma^{\downarrow} \sim e^{-S(2)+S(1)}$$

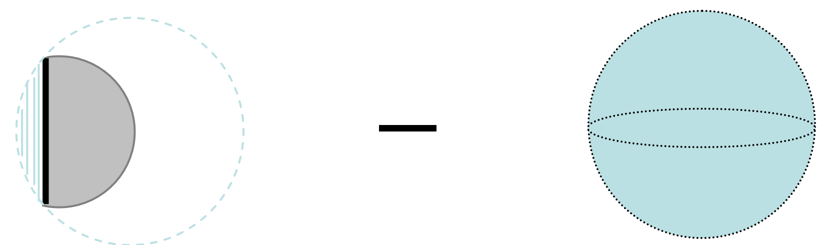
Entropy difference

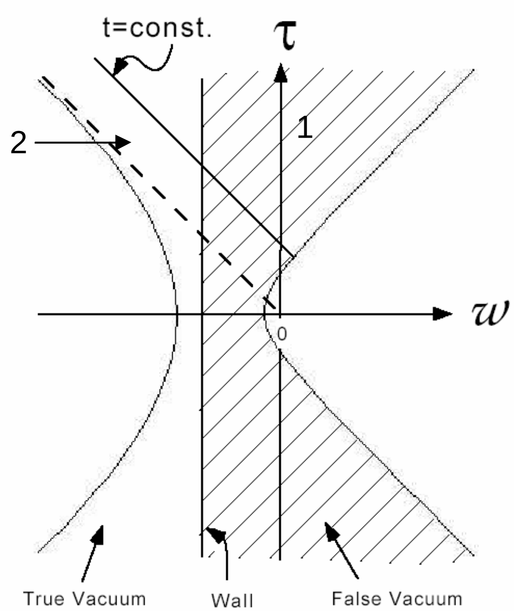


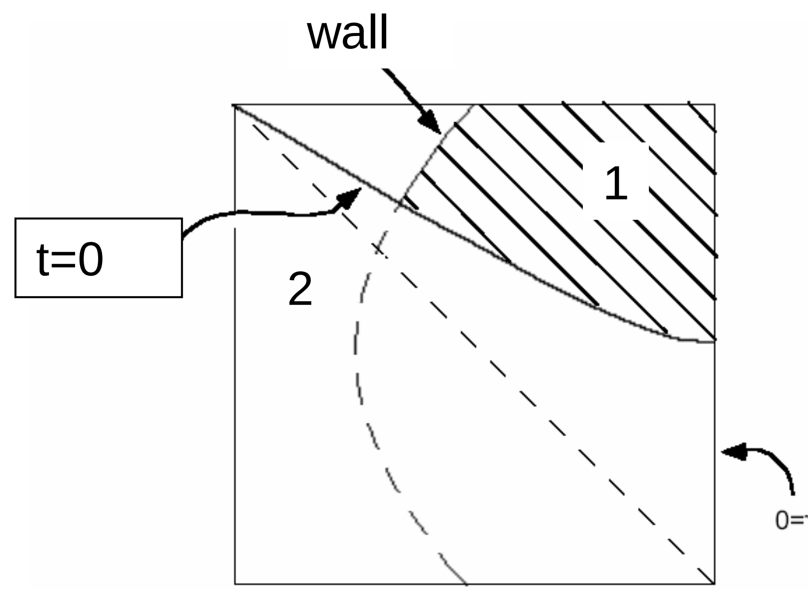
$B^{\downarrow} =$

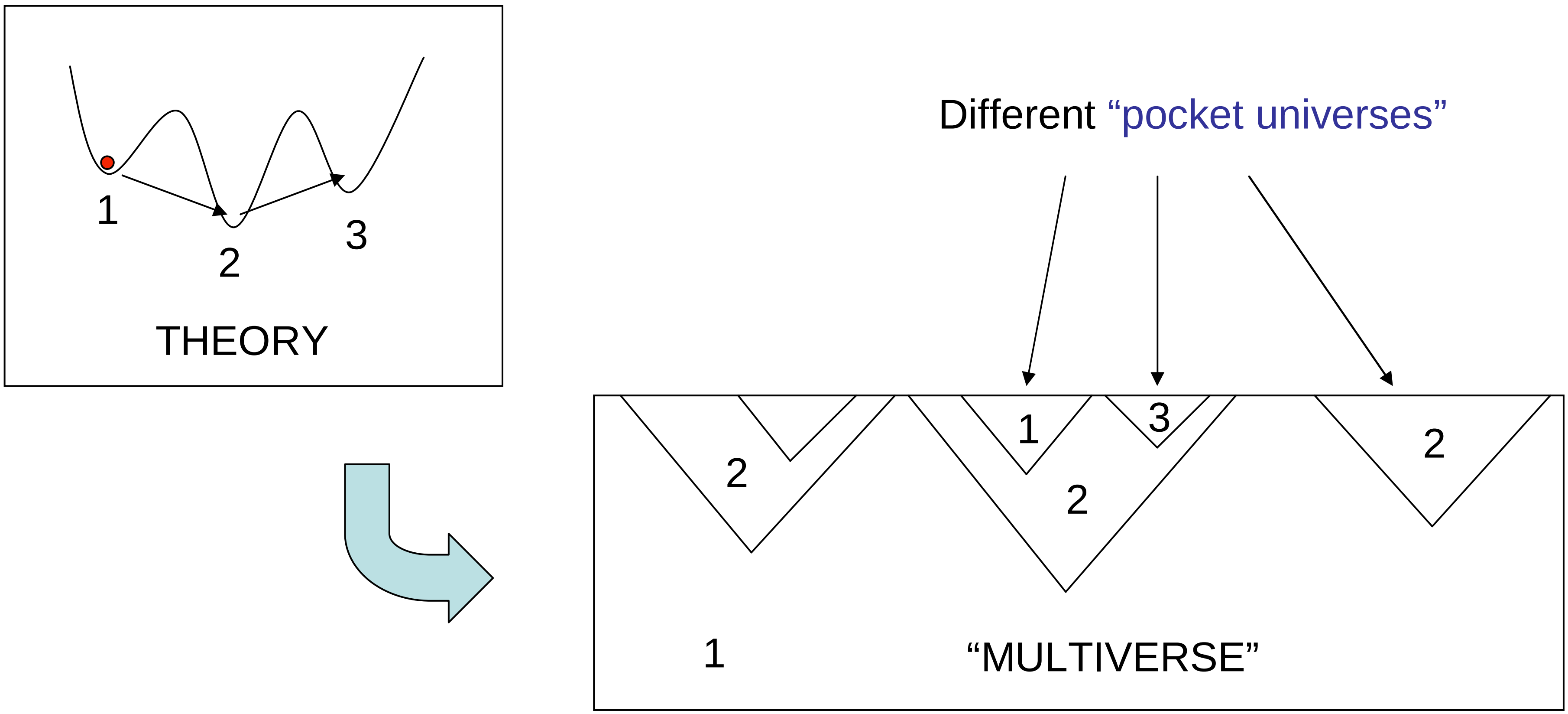


$B^{\uparrow} =$



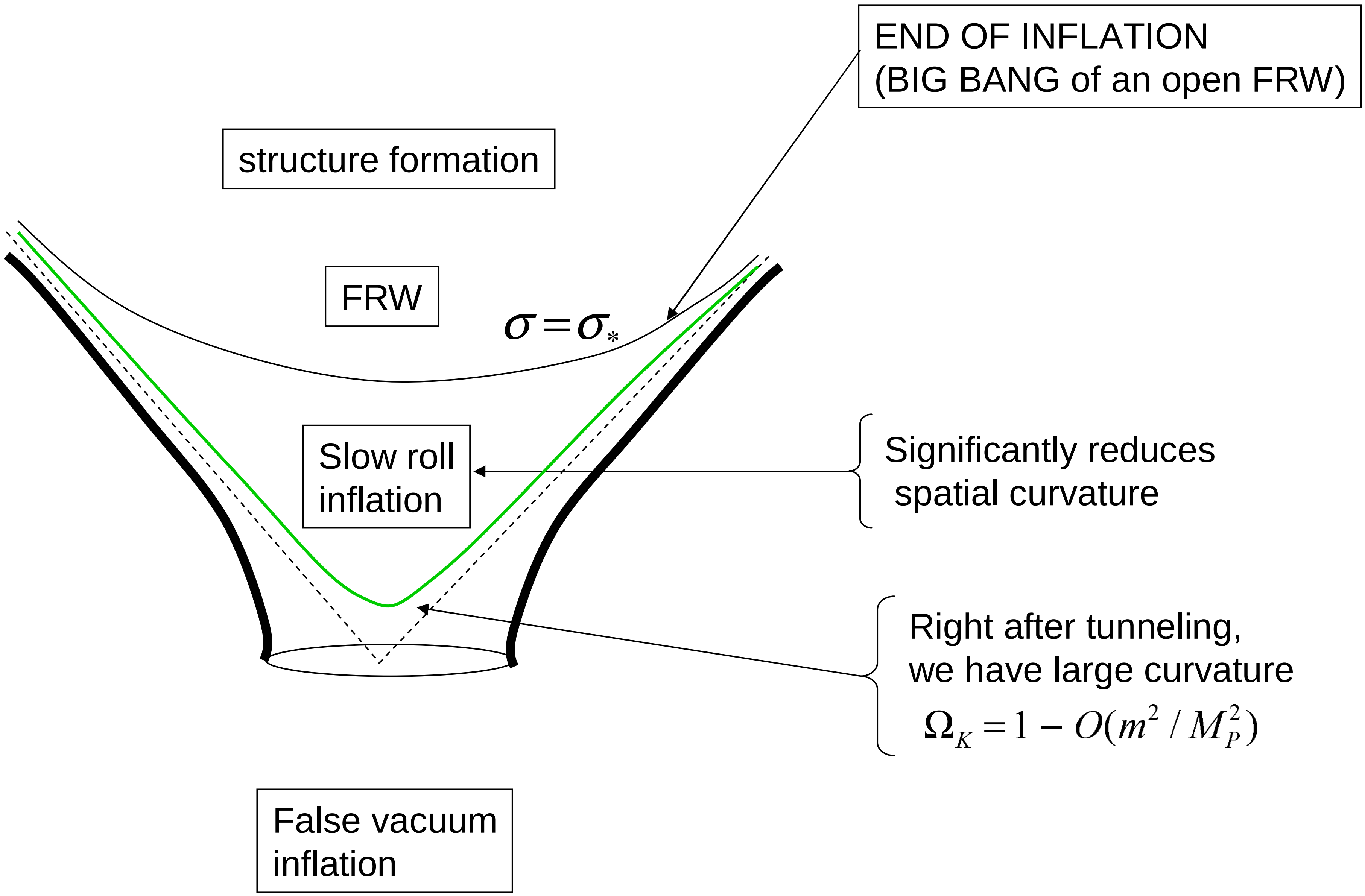






Can this picture be validated by observations?

Structure of a “pocket” universe



$$\Omega_K^* = e^{-2N_{\inf}}$$

$$e.g. \quad \Omega_K^{dec} \sim e^{-2N_{\inf}} \left(\frac{T_*}{T_{eq}}\right)^2 \left(\frac{T_{eq}}{T_{dec}}\right)$$

curvature “dilutes” as a^{-2}

OBSERVATIONS

$\Omega_K^{now} = -.01 \pm .01$

(WMAP + HST)

TAKE

$$|\,\Omega_K^{now}\,| < 0.02$$

(more stringent for negative K)

$$i.e. \quad |\,\Omega_K^{dec}\,| < 5\times10^{-5} \qquad (\ll 10^{-3})$$

COMPARE WITH ANTHROPIC BOUNDS
(curvature interferes with structure formation)

$$|\,\Omega_K^{dec}\,| \;\le\; 10^{-3} \text{ (for giant galaxies)}$$
$$10^{-2} \text{ (dwarf)}$$

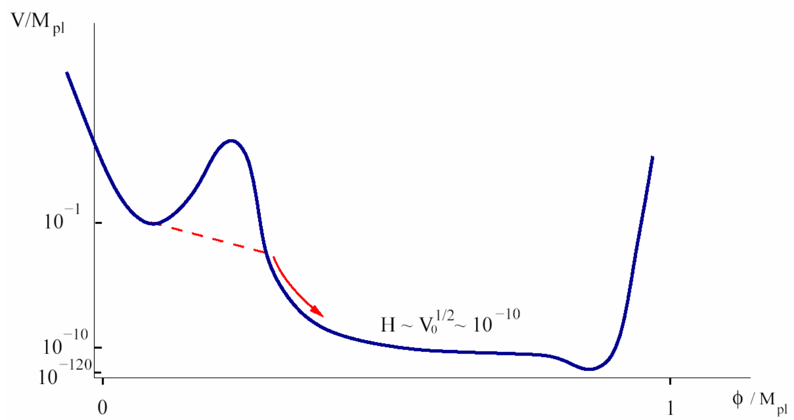
Vilenkin+Winitzki 97

$$\Rightarrow \quad N_{\inf}^{obs} - N_{\inf}^{Anth} > 1.5$$
$$2.5$$

Freivogel, Kleban, Rodriguez, Susskind, 05

WITH A BIT OF LANDSCAPING …

Freivogel, Kleban, Rodriguez, Susskind, 05



Integrating over length, height and slope of the plateau (with flat priors) and keeping $(\delta\rho / \rho) \sim (V^{3/2} / V')$ fixed,

$$dP(N) \propto N^{-4}\ dN$$

$$P(\Delta N < 2.5) \sim 3\Delta N / N \sim 10\%$$

…SATURATING THE OBSERVATIONAL BOUND DOESN'T LOOK SO UNLIKELY ANYMORE.

(there are, however, many however's)

What is it that we might expect to observe?

(Assuming Ω_K is close to its observational bound)

1- Signatures from the beginning of slow roll.
   Perhaps a smaller quadrupole than normal? (a “ post-diction”)

2- Signatures from BEFORE slow roll.

   a. Bubble wall fluctuations
   b. Gravity waves from the false vacuum phase
   c. Scalar perturbations from the false vacuum phase?

3- And, of course $\Omega_K \neq 0$

The wobbly bubble (and the waves it makes)

J.G.'96

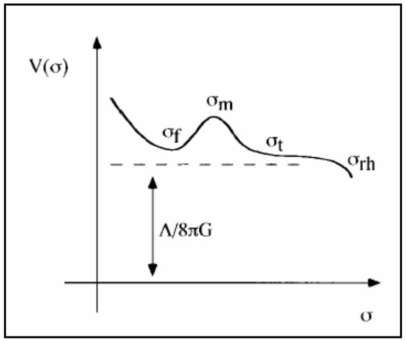
Consider a light bubble

μ wall tension

$R_0^{-1} \sim \Delta V \, \mu^{-1}$ wall acceleration

$G\mu \sim$ repulsive grav. acc.

assume
$$\begin{cases} G\mu \ll R_0^{-1} \qquad (G\mu^2 \ll \Delta V) \\ \Delta V \ll V \end{cases}$$



In this case, we can use field theory in external dS background

Region I

$$ds^2 = -dt^2 + a^2(t)d\Omega_{H^3}$$

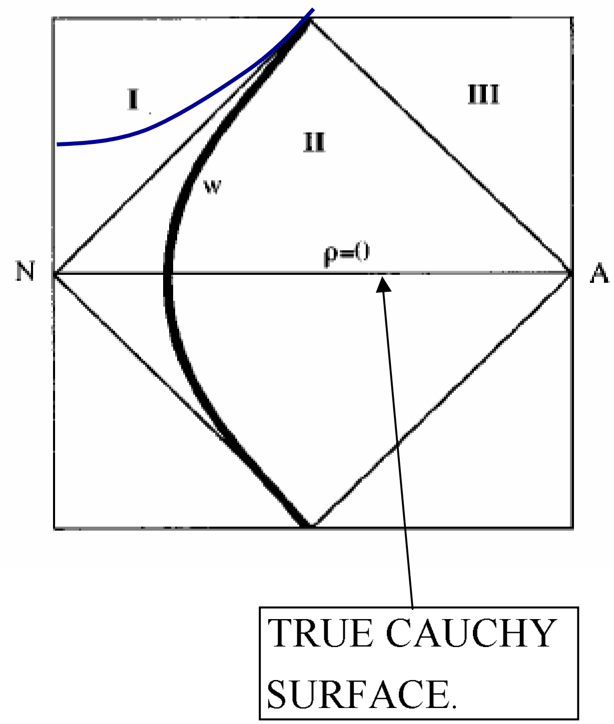
$$d\Omega_{H^3} = dr^2 + \sinh^2 r\ d\Omega^2$$

Region II

$$t \to i\tau$$
$$r \to \rho + i\pi/2$$

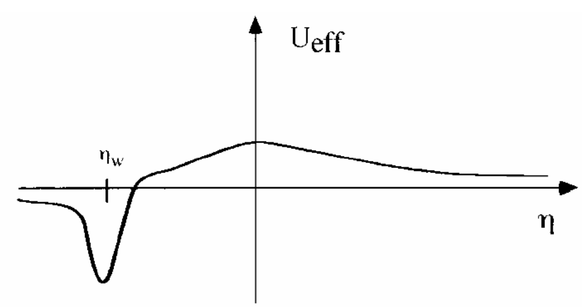
$$ds^2 = +d\tau^2 + R^2(\tau)d\Omega_{dS}$$

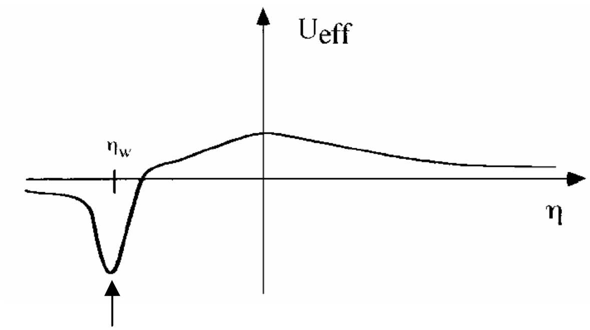
$$d\Omega_{dS} = -d\rho^2 + \cosh^2\rho\ d\Omega^2$$



$$\sigma(\tau, x^i) = \sigma_0(\tau) + \phi(\tau, x^i) \qquad \phi_{klm} = R^{-1}(\tau)F_k(\tau)Y_{klm}(x^i)$$

$$\begin{cases} [-{}^{(3)}\Box + k^2]Y_{klm}(x^i) \\ -\dfrac{d^2F_k}{d\eta^2} + R^2[m^2(\sigma_0) - 2H^2]F_k = (k^2 - 1)F_k \end{cases}$$





Holds a
bound state

$$[-{}^{(3)}\Box + k^2]Y_{klm}(x^i)$$

$$-\frac{d^2F_k}{d\eta^2} + R^2[m^2(\sigma_0) - 2H^2]F_k = (k^2-1)F_k$$

$$k^2 = -3$$

$$F_{-3} = N\sigma_0{}'(\eta) \quad \Longrightarrow \quad \sigma = \sigma_0(\tau + \overbrace{N\,Y_{-3lm}(x^i)}^{\text{wobble}})$$

What's with the odd looking tachyonic "mass" $k^2 = -3$? Just geometry

$$-{}^{(N)}\Delta = L(L+N-1)$$

Does it produce a catastrophic instability? Not really.

wobble proper size $\delta\tau \propto e^{\rho}$

bubble size $R_0 \cosh\rho$

Their ratio goes to a constant $\sim \dfrac{N}{R_0} = (\mu R_0^3)^{-1/2} \sim B^{-1/2}$

Non-sphericity is small when
Euclidean action is large

“scalar” transmutes into “tensor”

Inside the FRW

$$\left.\begin{aligned} \sigma &= \sigma_0(t+\delta t) \\ \delta t &= (\mu R_0)^{-1/2}\, Y_{-3lm}(r,\theta,\varphi) \end{aligned}\right\}$$

This is not normalizable on the FRW slices !!

Change to a
New gauge

$$\left\{\begin{aligned} t' &= t+\delta t \\ x^{i'} &= x^i - H\,\delta t^{,i} \end{aligned}\right. \Rightarrow \left\{\begin{aligned} \sigma &= \sigma_0(t') \\ h_{ij} &= \frac{H}{M_P}(G\mu R_0)^{-1/2}(Y_{|ij} - Y\gamma_{ij}) \end{aligned}\right.$$

Matter is smooth

LARGE
AMPLITUDE

Bona fide transverse,
traceless tensor
(at the bottom of the continuous spectrum
of normalizable tensor modes)

$$-\Delta h_{ij} = +3\ h_{ij}$$

Effect on CMB

$$\frac{\delta T}{T} \sim \int_0^{r_{ls}} h'_{rr}(r_{ls} - r)\, dr$$

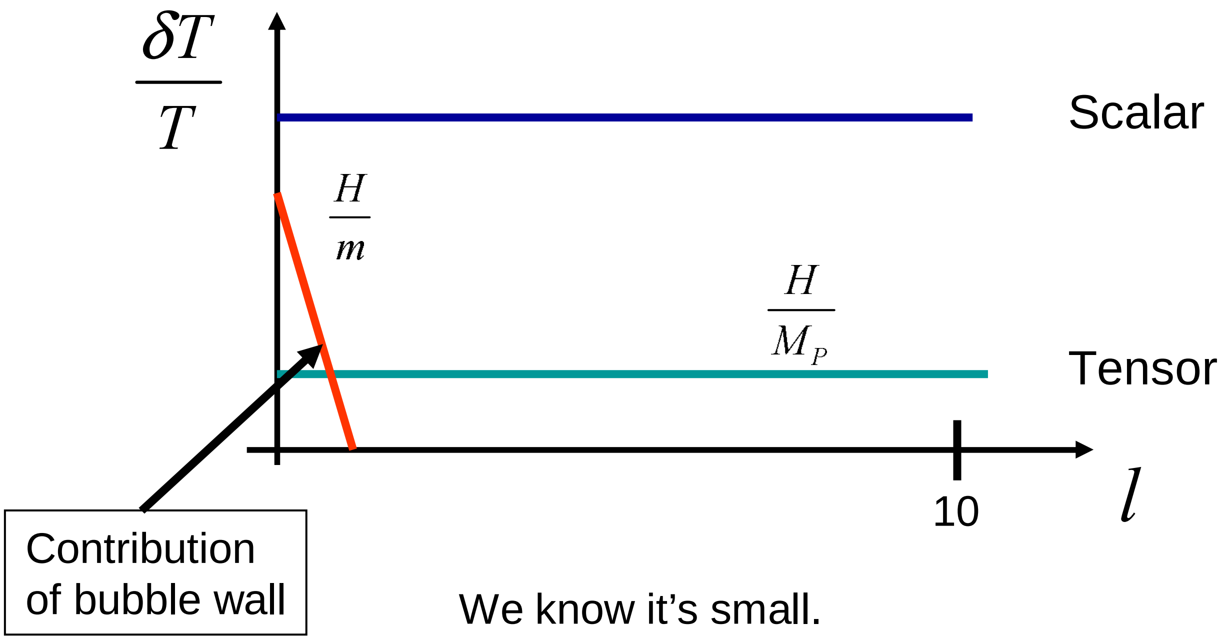
$$Y_{-3lm} \propto r^l \; Y_{lm}$$

$$r_{ls} \propto \Omega_K^{1/2}$$

$$\frac{\delta T}{T} \sim \frac{H}{M_P}\left(\frac{1}{G\mu R_0}\right)^{1/2} \Omega_K^{l/2}$$

$$\boxed{\Omega_K^{l/2} \sim 10^{-l}}$$

Exponentially decaying with multipole



We know it's small.
We may just hope it is not too small…

The full calculation

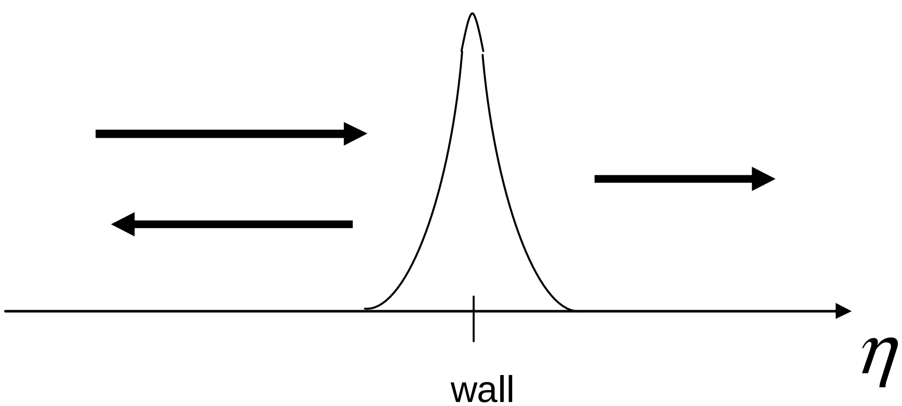
Sasaki, Tanaka Yakushige 97

J.G, Montes, Sasaki, Tanaka 98

Switching on the gravitational coupling to the bubble,
we can study the gravity waves in its presence.

$$-F'' + 4\pi G\,(\sigma'_0)^2\,F = k^2\,F$$

radial eqn. for the relevant gauge invariant variable



$$\Re(k) = \frac{(G\mu R_0)^2}{(G\mu R_0)^2 + k^2}$$

Reflection coefficient

$$P_h \propto GH^2 \left(1 - \frac{\Re(k)}{\cosh \pi k}\right)$$

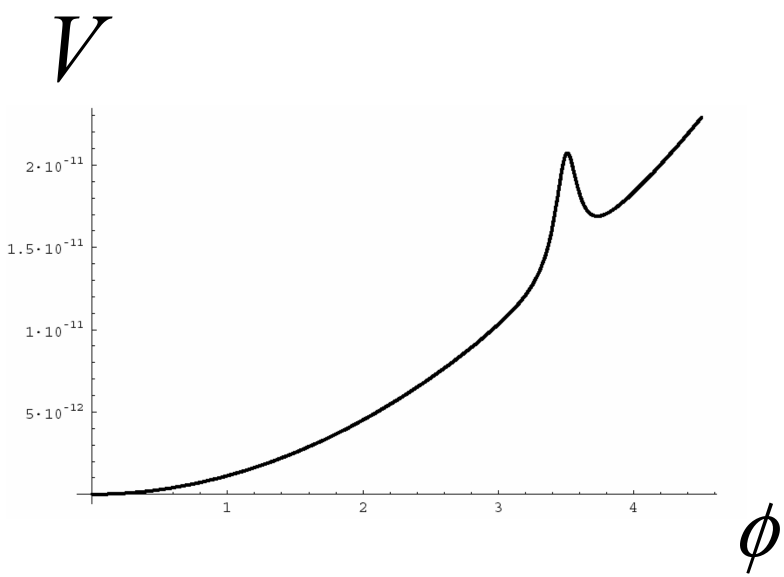
Reproduces the effect of wall fluctuation

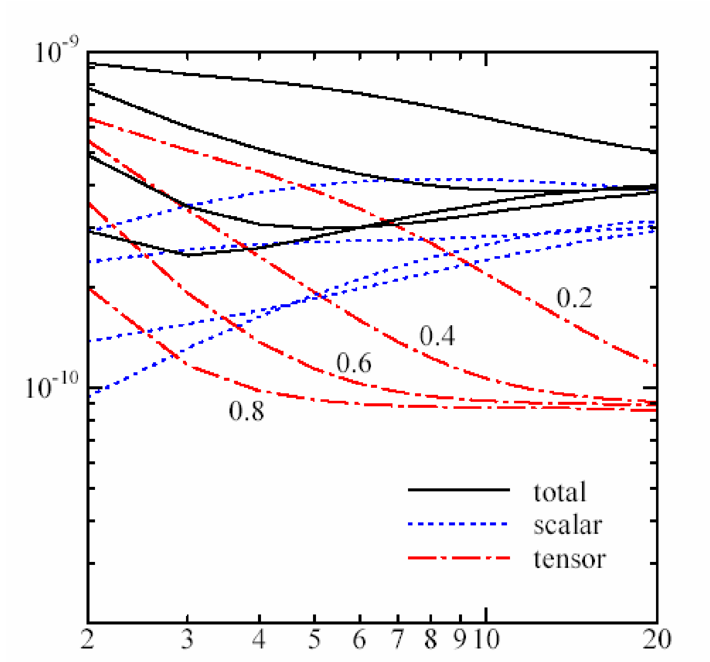
conclusions

- Signals can reach from before the bubble
- We haven’t seen any
- We may still see some in the future, if primordial gravity waves are found.
- It does not seem too likely, since the amount of slow roll has to be marginal. But the signature could be dramatic.
- As a question of principle, it might be interesting to quantify exactly how much information we can gather from the previous false vacuum phase.

- Holographic dual ? (Freivogel et al. 2006).

An example:





$$V(\phi) = \frac{m^2\phi^2}{2}\left(1 + \frac{\alpha^2}{\beta^2 + (\phi - v)^2}\right)$$

Linde, Sasaki, Tanaka 99