The Cosmological Constant Problems and Renormalization Group

Ilya Shapiro

DF-UFJF, Juiz de Fora, MG

Based on collaboration with:
Joan Solà, (Univ. de Barcelona, Spain)

Abstract.
The Cosmological Constant **(CC)** Problem emerges in Quantum Field Theory **(QFT)** in curved space-time due to the enormous magnitude of the induces vacuum energy.

After reviewing the CC problem we consider the possibility of a variable CC due to the quantum effects of matter fields. Assuming the standard (Appelquist & Carazzone) form of decoupling of massive fields at law energies we arrive at the cosmological models with the potentially observable predictions.

The history of the cosmological constant (CC) started when Einstein introduced the corresponding term into gravitational equations.

The original purpose was to provide static cosmological solution.

Nowadays we know Universe is expanding according to the Hubble law.
Why don't we remove the CC from the scene?

1) Acelerated expansion of the Universe has been detected in SN and CMB observations;

2) Positive vacuum energy is requested by anthropic considerations;

3) Independent vacuum CC is needed to achieve **renormalizable QFT in curved space**;

4) **Induced CC** (vacuum energy) emerges in the **SM** of particle physics;

5) **Possible variation** of the overall vacuum energy due to the quantum effects,
at both early and late stages of the evolution.

1) The **QFT in curved space-time** has been mainly developed in 70th and 80th. (see, e.g., books: Birrell, Davies ; Fulling ; Buchbinder, Odintsov, Sh.)

Renormalization & Renormalization Group

Renormalizable theory includes **classical action of vacuum** with, at least, following terms:

$$S_{vac} = S_{HE} + S_{HD}\,,$$

$$L_{HE} = -\frac{1}{16\pi G}\,R - \Lambda\,,$$

$$L_{HD} = a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\,,$$

$C^2 = C^2_{\mu\nu\alpha\beta}$ is the square of the Weyl tensor, $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is the Gauss-Bonnet integrand.

$G,\ \Lambda,\ a_{1,2,3,4}$ are **vacuum parameters**. On quantum level they are renormalized and become running constants, that is

depend on the energy scale.

Important notice:

Without vacuum parameter $\Lambda = \Lambda_v$ the theory is inconsistent.

Any loop of massive particle produces log. divergences of the Λ_v-type.

If $\Lambda_v \equiv 0$, these divergences can not be removed by renormalization.

What is the minimal possible range of Λ_v? At high energies it must be consistent with the renormalization group running.

$$(4\pi)^2 \, \mu \, \frac{d\Lambda}{d\mu} = \beta_\Lambda = \frac{m_s^4}{2} - 2m_f^4 \,, \qquad (1)$$

where μ is the typical energy of the external field (graviton, in the case).

Supersymmetry (SUSU) may provide $\beta_\Lambda = 0$, but at lower energy scales it is indeed broken.

The min. admissible value of Λ_v is m^4.
For neutrino it gives "correct" range
Sh. & Solà, Ph.L.B., 2001,

$$\Lambda \sim m_{\nu_1}^4 \sim 10^{-48} \, GeV^4 \,,$$

but this is perhaps just an accident.

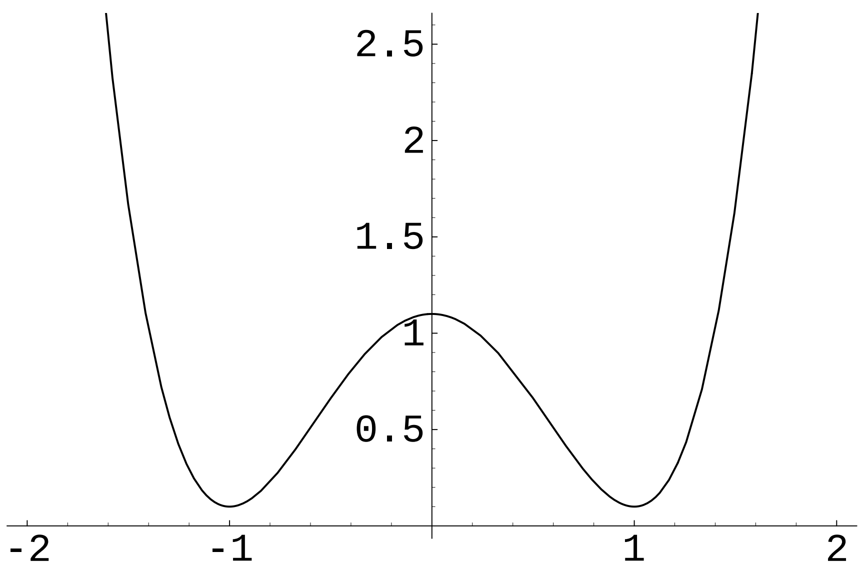
Induced CC. E.g. from **SSB:** In the stable point of the Higgs potential

$$V = -m^2\phi^2 + f\phi^4$$

we meet the VEV

$$\Lambda_{ind} = < V > \approx 10^8 \, GeV^4$$

(induced CC, Zeldovich, 1967).



Observed CC is a sum $\Lambda_{obs} = \Lambda_{vac} + \Lambda_{ind}$.

Λ_{vac} **is independent parameter** $\Longrightarrow$
it needs a special renormalization condition

$$\Lambda_{vac}(\mu_c) = \Lambda_{obs} - \Lambda_{ind}(\mu_c)\,,$$

μ_c is the energy scale where Λ_{obs} is measured.

So, the main CC relation is

$$\Lambda_{obs} = \Lambda_{vac}(\mu_c) + \Lambda_{ind}(\mu_c)\,.$$

Λ_{obs} is likely observed in SN-Ia experiments.

$$\Lambda_{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47\,GeV}.$$

The magnitudes of $\Lambda_{vac}(\mu_c)$ and $\Lambda_{ind}(\mu_c)$ are **55** orders greater than the sum, defining the precision of a necessary fine-tuning.

"Why they cancel so nicely" is the famous CC problem (review: S. Weinberg, 1989).

The origin of the problem is the difference between the M_F scale of Λ_{ind} and the μ_c scale of Λ_{obs}. It is a sort of hierarchy problem.

Further aspects:
1) The Universe is not static $\Rightarrow$ Λ could run.
2) Possible abrupt changes of the CC due to the phase transitions.

The universe was "prepared" beforehand in a special way, with a 55-order precision, such that today $\Lambda_{obs} \sim \rho_c$.

Can some symmetries solve it?

There were many attempts to solve the CC problems introducing more symmetries. The most remarkable example is SUSY.

But the CC problem emerges at very low energies, where SUSY is broken.
Typically, symmetries might solve the CC problem, but only at high energies, where the problem does not exist.

In the (super)string theory, the situation is even more complicated because the choice of the vacuum is not definite yet.

Even if the string vacuum would "indicate" zero CC, it is unclear how this can affect the low-energy physics.

At low energies, we know the appropriate method is QFT (especifically the SM, with SSB etc) and not a string theory.

Auto-relaxation mechanisms

There was a number of interesting attempts to create a sort of automatic mechanism for relaxing the CC (Abbot; Dolgov; Peccei, Solà, Wetterich; Hawking; Ford et al).

In 1989 Weinberg has discussed these approaches and has shown that they merely transfer CC fine tuning to other parameter(s). At the same time, it seems no comprehensive proof of this "no-go theorem" was given. Maybe one can modify Einstein equations in such a way that they do not "notice" CC.

Antropic considerations.

(Weinberg, Vilenkin, Garriga, Donoghue, ...)

This approach may be the most realistic, it also agrees with the QFT principles.

The idea is to study the limits on the CC and other parameters (e.g. neutrino mass) such that the cosmic structures (galaxies) are compatible with the human life.

The "shortcoming" is we never learn **why** the two counterparts of CC do cancel.

Renormalization Group (RG) solutions.
At low energies the quantum effects of some kind may produce an efficient screening of the observable CC.

1) IR quantum gravity (Polyakov, 1982, 2001), (Taylor & Veneziano, 1990.)
(Reuter et al, 2000),
(Tsamis & Woodard, 1995-2002).

2) IR quantum effects of the conformal factor - the $4d$ analog of the Polyakov model in $2d$ (Antoniadis and Mottola, 1992).

3) Driving the induced CC value between the GUT scale M_X and the cosmic scale μ_c by the quantum effects of finite GUT's. The level of fine-tuning for the vacuum CC is reduced (I.Sh., 1994, 2005). Numerical effect may require many copies of the fields. Also we have to be in a GUT vacuum, hence need some symmetry non-restoration mechanism.

4) Similar approach for the scalar fields. The values of the coupling grow up in the IR, increasing the effect (Jackiw et al, 2005).

Cosmological Constant Problems

• There are induced contributions to CC. Most of them are ~ 55 orders of magnitude greated than the observed sum.
Λ_{vac} is unique independent part of the CC.

• The main CC problem (I) is a hierarchy problem: the conflict between the SM scale $100\, GeV$ and the cosmic scale $\mu_c \sim 10^{-42}\, GeV$.

• The sub-leading quantum contributions to the induced CC depend on the details of the particle spectrum, hence the "solution" of the CC problem I is perhaps possible only after one can derive the SM spectrum from the **first principles**.

• The coincidence problem (II) is why $\Lambda_{obs} \propto \rho_c$ at the present epoch. The two problems are closely related.

We take a phenomenological point of view and don't try solving problems (I) & (II).

Instead we consider problem (III): whether CC may vary due to the IR quantum effects.

CC can vary due to the RG running?

At high energies scalar m_s and fermion m_f produce the RG equation

$$(4\pi)^2\,\mu\,\frac{d\Lambda}{d\mu} = \frac{m_s^4}{2} - 2m_f^4\,. \qquad (1)$$

In order to apply this to cosmology, we have to address two questions:

• **What is μ?**
•• **At which energy scale** (1) can be used?

Our answer to •: in the late Universe $\mu \sim H$.

The answer to ••: If applied to the late Universe, (1) results in a fast running of CC, breaking the standard cosmological model.

This does not happen, because in the QFT there is a phenomena called **decoupling**.

First investigation of decoupling for the case of gravity: E.Gorbar and I.Sh., JHEP 02(2003); 06(2003); 02(2004).

The QED example (flat space):

The 1-loop vacuum polarization is

$$-\frac{e^2\,\theta_{\mu\nu}}{2\pi^2}\int_0^1 dx\, x(1-x)\,\ln\frac{m_e^2+p^2x(1-x)}{4\pi\mu^2},$$

where $\theta_{\mu\nu}=(p_\mu p_\nu-p^2 g_{\mu\nu})$, μ is parameter of dimensional regularization.

$\beta^{\overline{\text{MS}}}$ is $\frac{e}{2}\mu\frac{d}{d\mu}$ acting on the formfactor of $\theta_{\mu\nu}$

$$\beta_e^{\overline{\text{MS}}}=\frac{e^3}{12\pi^2}.$$

β_e in the physical **mass-dependent scheme:** subtract at $p^2=M^2$ and take $\frac{e}{2}M\frac{d}{dM}$.

The UV limit ($M\gg m_e$): $\beta_e=\beta_e^{\overline{\text{MS}}}$.

The IR limit ($M\ll m_e$):

$$\beta_e=\frac{e^3}{60\pi^2}\cdot\frac{M^2}{m_e^2}+\mathcal{O}(\frac{M^4}{m^4}).$$

Appelquist & Carazzone, (1975)

Compared to $\beta_e^{UV}=\beta_e^{\overline{\text{MS}}}$, in the IR there is a **suppression of the form** p^2/m_e^2.

Does decoupling take place in curved space?

Main physical motivations:

• The CC running.

•• Modified Starobinsky model of inflation.

We need an explicit form of decoupling!

Massive scalar field:

$$S_s = \frac{1}{2}\int d^4x\, g^{1/2}\,\Big\{g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2 + \xi R\varphi^2\Big\}\,.$$

Euclidean Effective Action is

$$\Gamma[g_{\mu\nu}] = -\frac{1}{2}\,\mathrm{Tr}\,\ln\Big(-\nabla^2 + m^2 + \xi R\Big)\,.$$

Weak point: we do not have covariant version of the mass-dependent scheme of renormalization.

We can perform calculations only on the flat background (linearized gravity)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The $\Gamma[g_{\mu\nu}]$ terms of lowest order in $h_{\mu\nu}$ are gr quadratic, hence we need the bilinear in $h_{\mu\nu}$ corrections to the *graviton propagator.*

The polarization operator must be compared to the tensor structure of the Lagrangians

$$L_{HE} = -\frac{1}{16\pi G}\,(R + 2\Lambda) \qquad \text{and}$$

$$L_{HD} = a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\,.$$

In the physical mass-dependent scheme, in the framework of linearized gravity we obtain

$$\beta_1 \;=\; -\,\frac{1}{(4\pi)^2}\left(\frac{1}{18a^2} - \frac{1}{180} - \frac{a^2-4}{6a^4}\,A\right)\,.$$

$$A = 1 + \frac{1}{a}\,\ln\left|\frac{2-a}{2+a}\right| \quad \text{and} \quad a^2 = \frac{4\Box}{4m^2-\Box}\,.$$

UV and IR Limits:

$$\beta_1^{UV} = -\frac{1}{120\,(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right)\,,$$

$$\beta_1^{IR} = -\,\frac{1}{1680\,(4\pi)^2}\cdot\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right)\,.$$

Appelquist & Carazzone theorem
for the gravitational vacuum sector.

For a_3, a_4 the situation is similar.

Big Problem:

In the **perturbative** (in $h_{\mu\nu}$) approach we do not observe RG running for the cosmological and inverse Newton constant.

Why did we get $\beta_\Lambda = \beta_{1/G} = 0$?

Running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ formfactors into effective action.

Say, in QED:

$$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\, F_{\mu\nu}\, \ln\Big(-\frac{\Box}{\mu^2}\Big)\, F^{\mu\nu}\,.$$

Similarly, in gravity it is possible to insert formfactors $f(\Box)$ into the C^2 and'' R^2 terms, providing their RG running.

However, such insertion **is not possible** for the CC and $1/G$, since $\Box\Lambda = 0$ and $\Box R$ is a full derivative.

Possible non-local terms, e.g.

$$m^4 \int d^4x\, g^{1/2}\, R_{\mu\nu} \left(-\frac{\mu^2}{\Box}\right)^2 R^{\mu\nu}$$

do not show up in this framework.

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$?

Probably not. Physical β_Λ and $\beta_{1/G}$ should tend to the corresponding $\beta^{\overline{\mathrm{MS}}}$-functions in the UV limit.

Perhaps the **linearized gravity approach is not appropriate for the CC and Einstein terms.**

What would happen if the decoupling for the CC is the standard one?

I.Sh., J.Solà, Nucl.Phys.(PS), IRGA-2003 127B.

I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, Phys. Lett. B574 (2003).

I.Sh., J.Solà, H.Stefancic, JCAP 0501 (2005).

Partially based on the ideas of:

I.Sh., J.Solà, JHEP 02 (2002);

A.Babic, B.Guberina, R.Horvat, H.Stefancic, PRD 65 (2002).

We suppose a standard AC-like decoupling for each particle with the mass m:

$$\beta_\Lambda^{IR}(\mu) \sim \frac{\mu^2}{m^2}\, \beta_\Lambda^{UV}\,.$$

In general, $\overline{\text{MS}}$ contribution should be multiplied by some function $F(\mu/m)$. In the UV region $F(\mu/m) \to 1$.

In the IR $\mu \ll m$ one can expand

$$F(\mu/m) = \sum_{n=1}^{\infty} k_n\, (\mu/m)^{2n}\,.$$

Our hypothesis is that $k_1 \neq 0$.

This **phenomenological input** does not contradict any known law of physics. At the present state of knowledge its validity can be checked only via **comparison with the experimental data.**

The scale difference between H and the lightest neutrino is 10^{-30},
with QCD scale 10^{-40},
between H and M_P is 10^{-60}.
All mass scales decouple the same way!

Assume $\mu \sim H$, then $\beta_{\Lambda}^{IR} \sim \frac{H^2}{m^2}\, \beta_{\Lambda}^{UV}$.

- **Main point:** $\beta_{\Lambda}^{UV}(m) \propto m^4$;

- All massive particles provide additive contributions to β_{Λ};

All in all, we arrive at

$$\beta_{\Lambda} = \frac{1}{(4\pi)^2}\, \sigma\, M^2\, H^2\,,$$

where M is unknown **mass parameter** and $\sigma = \pm 1$ depending on whether fermions or bosons dominate **at the highest energies.**

$$\Lambda(H) = \Lambda_0 + \frac{\sigma\, M^2}{8\pi^2}\left(H^2 - H_0^2\right)\,,$$

where Λ_0 is the CC at the present epoch when the Hubble parameter is H_0.

If we assume that the particles spectrum goes to the Planck scale,

$$M^2 = M_P^2 \implies |\beta_{\Lambda}| \sim 10^{-47}\, GeV^4\,,$$

close to the SN & CMB data on the CC.

We link the cosmological model with variable Λ and the particle mass spectrum.

Cosmological model based on running CC.
For simplicity $k = 0$ case.

Along with RG, there is Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) \ ,$$

$\rho = \rho_R + \rho_M$, and the conservation law. Its precise form is not obvious.
The most elaborated version (p - pressure)

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0 \, ,$$

Even for the equation of state $p = \alpha\rho$, the solution is completely analytical. In terms of the red-shift $z = a_0/a - 1$

$$\rho(z;\nu) = \rho_0 (1+z)^r \qquad (1)$$

and

$$\Lambda(z;\nu) = \Lambda_0 + \frac{\nu}{1-\nu}\left[\rho(z;\nu) - \rho_0\right] \, , \qquad (2)$$

where ρ_0, Λ_0 are present day values, and

$$\nu = \frac{\sigma M^2}{12\pi M_P^2} \qquad \text{and} \qquad r = 3(1-\nu)(\alpha+1) \, .$$

At $\nu \to 0$ we recover the standard result for $\Lambda = const$.

Test: **nucleosynthesis epoch** $\rho_{\text{rad}} \gg \rho_{\text{mat}}$.

$$\rho_{rad}(T) = \frac{\pi^2 g_*}{30} T^4 \left(\frac{T_0}{T}\right)^{4\nu}$$

$T_0 \simeq 2.75\,K$ is present CMB temperature.

For $\nu \geq 1$ the $\rho_{rad}(T)$ would be constant.
In order not to be ruled out by the nucleosynthesis:

$$|\,\Lambda_R\,/\,\rho_R\,| \simeq |\,\nu\,/\,(1-\nu)\,| \simeq |\nu| \ll 1\,.$$

A nontrivial range is, e.g., $0 < |\nu| \leq 0.1$.
Both signs of ν are allowed.

The "canonical" choice $M^2 = M_P^2$ gives

$$|\nu| = \frac{1}{12\,\pi} \simeq 2.6 \times 10^{-2}\,.$$

The nucleosynthesis constraint is consistent with the effective approach.

Zero CC in the remote future $\sim\ \nu \approx 0.7$

Marginal value for the nucleosynthesis!

Whether the permitted values $\nu \ll 1$ may lead to **observable consequences?**
The remarkable answer is: **yes.**

Consider the "recent" Universe $0 < z \leq 2$.
The relative deviation

$$\delta_\Lambda(z;\nu) = \frac{\Lambda(z;\nu) - \Lambda_0}{\Lambda_0}$$

The existing estimates for the CC from the SN data correspond to some $z = z_0$. Taking $z_0 \simeq 0.5$, we find $\delta_\Lambda(z = 1.5; \nu_0) = 14$.

However! Perturbations in the model

$$\Lambda = \Lambda_0 + const \cdot (H^2 - H_0^2)$$

were investigated using $V(\varphi)$ analog model
R. Opher, A. Pelinson, PRD 70 (2004).

The limit $|\nu| < 10^{-6}$, that corresponds to the gap between M_X and M_P .

Similar bound comes from direct evaluation of linear perturbations
J. Fabris, J. Solà and I.Sh. (in preparation).

From the field-theoretical side, the existance of covariant, matter independent, effective action of vacuum puts under question the consevation law involving both Λ and ρ .

Another conservation law: $\nabla^\mu(G\tilde{T}_{\mu\nu}) = 0$, or

$$\frac{d}{dt}\,[G(\Lambda+\rho)] + 3\,G\,H\,(\rho+p) = 0\,,$$

together with

$$\dot{\rho} + 3\,H\,(\rho+p) = 0$$

gives (in this way we admit G may vary.)

$$(\rho+\Lambda)\,\dot{G} + G\,\dot{\Lambda} = 0\,.$$

I.Sh., J.Solà, H.Stefancic, JCAP 0501 (2005).

Full set of equations includes also

$$\rho+\Lambda = \frac{3H^2}{8\,\pi\,G}\,, \qquad \Lambda = C_0 + C_1\,H^2\,.$$

Solution for $G(H;\nu)$ is

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}\,,$$

where $G(H_0) = G_0 \equiv 1/M_P^2$.

In the cosmological setting we idintify the RG parameter μ with the Hubble parameter H

The uniqueness of the effective action indicates that the dependence

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}$$

may manifest itself also in other applications. For example, within the astrophysical setting we have to identify

$$\frac{\mu}{\mu_0} \sim \frac{r_0}{r}\,,$$

where r_0 is a reference galactic length of the visible part of the galaxy.

Then we arrive at the slow running $G(r)$

$$G(r) = \frac{G_0}{1+\nu\,\ln\left(r_0^2/r^2\right)}\,.$$

This is indeed the form suggested long ago

T. Goldman et al., Ph.L.B 281 (1992);
O. Bertolami et al., Ph.L.B 311 (1993).

on the basis of higher derivative quantum gravity RG and no-decoupling hypothesis.

Conslusions:

• One can consider the possibility of variable CC due to the quantum vacuum effects of matter fields indepenent on the difficulties with solving the CC problems I (Fine-tuning) and II (coincidence).

• Phenomenological estimates show that the quantum matter effects **may** provide the "running" of the observable CC in the range compatible with the constraints coming from perturbations.

• The quadratic decoupling for the CC looks the only one possible form compatible with covariance. It leads to soluble cosmological models. ν represents the unique arbitrary parameter of this model.

• In the $\Lambda(\mu,\nu)$, $G(\mu,\nu)$ model arrived at the astrophysical prediction.
$G(r)$ is compatible with observations
for $\nu \sim 10^{-6}$ (in the leading approximation).
This value corresponds to the GUT's particle spectrum and is likely to be compatible with the cosmic perturbations bounds.