

Renormalized Kaluza-Klein theories.

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$$S = \int d^D x L(\Phi_i)$$

Extra dimensions form a Compact manifold $M = \frac{1}{R}$

Physical intuition: physics at short distances should become D -dimensional, even if it is four-dimensional for $E \ll M$.

TWO ALTERNATIVES:

- D-dimensional one loop renormalization followed by mode expansion $\Phi(x, \theta) = \sum \Phi_n(x) Y_n(\theta)$

OR ELSE

- One can start from the beginning from the mode expansion $S = \int d^4 x L(\Phi_n)$ and then renormalize the four-dimensional lagrangian corresponding to the full KK tower.

ARE THESE TWO PROCEDURES EQUIVALENT?

$$[M, R] = 0 ?$$

Background Field

$$\Phi(x) = \bar{\phi}(x) + \phi(x)$$

$$\Psi(x) = \tilde{\psi}(x) + \psi(x)$$

$$S = \int d^n x \left(\frac{1}{2} \phi_M A_{MN} \phi_N + \bar{\psi} B \psi + \phi_N \bar{\Gamma}_N \psi + \bar{\psi} \Gamma_M \phi_M \right)$$

One-loop effective action given by a (super)det.

$$W \equiv \bar{S} + \frac{1}{2} \log \text{sdet } M$$

$$M = \begin{pmatrix} M_{++} & M_{+-} \\ M_{-+} & M_{--} \end{pmatrix}$$

$$\text{ber } M \equiv \text{sdet } M \equiv \det M_{++} \det^{-1} (M_{--} - M_{-+} M_{++}^{-1} M_{+-})$$

Γ_∞ given by Heat kernel

$$\begin{aligned}\log \det \Delta &= - \int \frac{d\tau}{\tau} e^{-\tau \Delta} \\ &= - \int_0^\infty \frac{d\tau}{\tau^{1+n/2}} \sum_{p=0} a_p \tau^{p/2}\end{aligned}$$

Short time (UV) expansion: de Witt-Schwinger

Integrals diverge at the lower end.

Besides, $a_p = \int d^n x b_p[\bar{\phi}]$.

Two possible ways of regulating the proper time integrals:

- **Dimensional regularization:** Only one term survives.

$$\log \det \Delta|_{div} = \frac{2\mu^\epsilon}{\epsilon} a_n(\Delta).$$

- **Proper time cutoff:** $\epsilon \equiv \Lambda^{-1}$.

$$\log \det \Delta|_{div} = \frac{1}{3}a_0\Lambda_{(d=6)}^6 + \frac{1}{2}a_2\Lambda_{(d=6)}^4 + a_4\Lambda_{(d=6)}^2 + a_6 \log \frac{\Lambda_{(d=6)}^2}{\mu_{(d=6)}^2}$$

$$\log \det \Delta|_{div} = \frac{1}{2}a_0\Lambda_{(d=4)}^4 + a_2\Lambda_{(d=4)}^2 + a_4 \log \frac{\Lambda_{(d=4)}^2}{\mu_{(d=4)}^2}$$

- **This can also be applied to odd-dimensional manifolds.**

In terms of the compactification scale

$$d^6 x = d^4 x \frac{d^2 \theta}{M^2}$$

Logarithmic divergences would match if

$$\frac{\Lambda_{(d=6)}^2}{M^2} \equiv \log \frac{\Lambda_{(d=4)}^2}{\mu_{(d=4)}^2}$$

(when $M \rightarrow \infty$)

Even that is not exactly true

$$a_4^{(d=6)} \neq a_4^{(d=4)}$$

QED₆ in $\mathbb{R}^4 \times \mathbb{T}^2$

Quadratic divergences:

$$a_4 = \int \frac{d^6 x}{(4\pi)^3} \left(\frac{4}{3} e^2 \bar{F}_{MN}^2 + 4e^2 \bar{\eta} \not{D} \eta + 12m e^2 \bar{\eta} \eta \right)$$

(6D canonical dimensions:)

$$e_{(d=6)}^2 \equiv \frac{e^2}{M^2}$$

$$\psi_{(d=6)} = M \psi$$

$$A_{\mu}^{(d=6)} = M A_{\mu}$$

**Logarithmic divergences
(The six-dimensional counterterm in d.r.)**

$$\begin{aligned}
 a_6 = \int \frac{d^6 x}{(4\pi)^3} & \left(-\frac{1}{12} e^4 \bar{\eta} \Sigma_{MNL} \eta \bar{\eta} \Sigma^{MNL} \eta + \right. \\
 & \frac{19}{15} e^2 m \bar{\eta} \bar{D}_M \bar{D}^M \eta + \frac{2}{15} e^3 \bar{\eta} \gamma_N \bar{D}_M \eta \bar{F}^{MN} - \\
 & - e^3 m \bar{\eta} \gamma_M \gamma_N \eta \bar{F}_{MN} - 2e^2 m^2 \bar{\eta} \gamma^M \bar{D}_M \eta - \\
 & 6e^2 m^3 \bar{\eta} \eta - \frac{11}{45} e^2 \bar{D}_R \bar{F}_{MN} \bar{D}^R \bar{F}^{MN} + \\
 & \left. + \frac{23}{9} e^2 \bar{D}_M \bar{F}^{MN} \bar{D}^R \bar{F}_{RN} - \frac{4}{3} e^2 m^2 \bar{F}_{MN} \bar{F}^{MN} \right)
 \end{aligned}$$

The four dimensional viewpoint

Harmonic expansion

$$\phi(x, y) = \frac{1}{2\pi\sqrt{R_5 R_6}} \sum_{n_1, n_2} \phi_{n_1 n_2}(x) e^{i\left(\frac{n_1 y_1}{R_1} + \frac{n_2 y_2}{R_2}\right)}$$

$$\equiv \frac{1}{2\pi\sqrt{R_5 R_6}} \sum_n \phi_n(x) e^{i\frac{n}{R} \cdot y}$$

The four dimensional theory.

$$\begin{aligned}
 S = \int d^4x \sum_{n_5, n_6} & \left(\bar{\psi}_n^1 \partial \psi_n^1 + \bar{\psi}_n^2 \partial \psi_n^2 + \right. \\
 & \bar{\psi}_n^1 \left(i \frac{n_5}{R_5} + \frac{n_6}{R_6} \right) \psi_n^2 - \bar{\psi}_n^2 \left(i \frac{n_5}{R_5} - \frac{n_6}{R_6} \right) \psi_n^1 + \\
 & + m \left(\bar{\psi}_n^1 \psi_n^1 - \bar{\psi}_n^2 \psi_n^2 \right) \\
 & - \frac{1}{2} (A_\mu^n)^* \left(\square - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A_\mu^n - \\
 & \frac{1}{2} (A_5^n)^* \left(\square - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A_5^n - \\
 & - \frac{1}{2} (A_6^n)^* \left(\square - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A_6^n - \\
 & e \sum_m \left(\bar{\psi}_m^1 A_{m-n} \psi_n^1 + \bar{\psi}_m^2 A_{m-n} \psi_n^2 + \right. \\
 & \quad \left. + \bar{\psi}_m^1 A_5^{m-n} \psi_n^2 \right. \\
 & \quad \left. - \bar{\psi}_m^2 A_5^{m-n} \psi_n^1 - i \bar{\psi}_m^1 A_6^{m-n} \psi_n^2 - i \bar{\psi}_m^2 A_6^{m-n} \psi_n^1 \right)
 \end{aligned}$$

Gauge symmetry

$$\delta A_\mu^n = i\partial_\mu \Lambda_n$$

$$\delta A_5^n = -\frac{n_5}{R_5} \Lambda_n$$

$$\delta A_6^n = -\frac{n_6}{R_6} \Lambda_n$$

SSB of 6-d Lorentz

$$O(1, 5) \rightarrow O(1, 3) \times O(2) \times O(2)$$

Zero mode part

$$A_5^0 - iA_6^0 \equiv \phi^0 \equiv \phi$$

$$\begin{aligned} S_{zm} = & \int d^4x \left(\bar{\psi}^1 \not{\partial} \psi^1 + \bar{\psi}^2 \not{\partial} \psi^2 + m (\bar{\psi}^1 \psi^1 - \bar{\psi}^2 \psi^2) \right. \\ & - \frac{1}{2} A_\mu \square A^\mu - \frac{1}{2} \phi^* \square \phi \\ & \left. - e (\bar{\psi}^1 \not{A} \psi^1 + \bar{\psi}^2 \not{A} \psi^2 + \bar{\psi}^1 \phi \psi^2 - \bar{\psi}^2 \phi^* \psi^1) \right) \end{aligned}$$

Ward Identities

The six-dimensional WI

$$\frac{1}{\alpha} \square \partial_M A^M + \partial_M \frac{\delta \Gamma_D}{\delta A_M} + e \psi \frac{\delta \Gamma_D}{\delta \psi} - e \bar{\psi} \frac{\delta \Gamma_D}{\delta \bar{\psi}} = 0$$

The four-dimensional WI

$$\begin{aligned} & \frac{1}{\alpha} \left((\square - |N|^2) \left(i \partial_\mu A_{-n}^\mu + \frac{n_5}{R_5} A_5^{-n} + \frac{n_6}{R_6} A_6^{-n} \right) - \right. \\ & \quad \left. - \frac{n_5 n_6}{R_5 R_6} \left(\frac{n_5}{R_5} A_6^{-n} + \frac{n_6}{R_6} A_5^{-n} \right) + i \partial_\mu \frac{\delta \Gamma_4}{\delta A_\mu^n} \right. \\ & \quad \left. + \frac{n_5}{R_5} \frac{\delta \Gamma}{\delta A_5^n} + \frac{n_6}{R_6} \frac{\delta \Gamma}{\delta A_6^n} + \right. \\ & \quad \left. + i e \sum_m \left(\psi_{m-n}^1 \frac{\delta \Gamma}{\delta \psi_m^1} - \psi_{m-n}^2 \frac{\delta \Gamma}{\delta \psi_m^2} - \bar{\psi}_{n-m}^1 \frac{\delta \Gamma}{\delta \bar{\psi}_m^1} + \right. \right. \\ & \quad \quad \left. \left. \bar{\psi}_{n-m}^2 \frac{\delta \Gamma}{\delta \bar{\psi}_m^2} \right) \right) = 0 \end{aligned}$$

Zero mode 4-d counterterms

$$a_4^{(zm)} = \int \frac{d^4x}{(4\pi)^2} \left(\frac{4}{3} e^2 \bar{F}_{\mu\nu}^2 - 4e^2 \bar{\phi}^* \square \bar{\phi} + 8e^2 m^2 |\bar{\phi}|^2 - 4e^4 |\bar{\phi}|^4 + 4e^2 (\bar{\eta}^1 \bar{D}\eta^1 + \bar{\eta}^2 \bar{D}\eta^2) + 12me^2 (\bar{\eta}^1 \eta^1 - \bar{\eta}^2 \eta^2) + 8e^3 \bar{\eta}^2 \bar{\phi}^* \eta^1 - 8e^3 \bar{\eta}^1 \bar{\phi} \eta^2 \right)$$

The scalar $\bar{\phi}$ is not protected by the four-dimensional WI

Full KK tower 4-d counterterm

$$\begin{aligned}
a_4 = & \int \frac{d^4x}{(4\pi)^2} \sum_l \left((-4m^4 + 2|L|^4 - 8m^2|L|^2) \right. \\
& + \frac{4}{3}e^2 \sum_n \bar{F}_{\mu\nu}^n \bar{F}_{-n}^{\mu\nu} - 4e^2 \sum_n \bar{\phi}_n^* \square \bar{\phi}_{-n} - \\
& - 8e(m^2 + |L|^2) (L\bar{\phi}_0^* - L^*\bar{\phi}_0) + \\
& 8e^2 \sum_n (m^2 + |L + N|^2 + |N|^2) \bar{\phi}_n^* \bar{\phi}_{-n} - \\
& - 4e^2 \sum_n (N^* + L^*) L^* \bar{\phi}_n \bar{\phi}_{-n} - \\
& 4e^2 \sum_n (N + L) L \bar{\phi}_n^* \bar{\phi}_{-n}^* + \\
& + 8e^3 \sum_{m,n} \bar{\phi}_{m-l}^* \bar{\phi}_{l-n} (M\bar{\phi}_{n-m}^* - N^*\bar{\phi}_{n-m}) + \\
& 4e^2 \sum_{m,n,s} \bar{\phi}_{m-l}^* \bar{\phi}_{l-s} \bar{\phi}_{s-n}^* \bar{\phi}_{n-m} - \\
& - 4e^2 \sum_n N^* \partial_\mu \bar{\phi}_n \bar{A}_{-n}^\mu + 4e^2 \sum_n N \partial_\mu \bar{\phi}_n^* \bar{A}_{-n}^\mu +
\end{aligned}$$

$$\begin{aligned}
& 4e^2 \sum_n |N|^2 \bar{A}_\mu^n \bar{A}_{-n}^\mu + \\
& + 8e^2 \sum_{n \neq 0} (\bar{\eta}_{l-n}^1 \partial \eta_{l-n}^1 + \bar{\eta}_{l-n}^2 \partial \eta_{l-n}^2) - \\
& 8e^3 \sum_{m \neq 0, n} (\bar{\eta}_{l-m}^1 \bar{A}_{l-n} \eta_{n-m}^1 + \bar{\eta}_{l-m}^2 \bar{A}_{l-n} \eta_{n-m}^2) + \\
& + 24me^2 \sum_{n \neq 0} (\bar{\eta}_{l-n}^1 \eta_{l-n}^1 - \bar{\eta}_{l-n}^2 \eta_{l-n}^2) + \\
& 16e^3 \sum_{m \neq 0, n} \bar{\eta}_{l-m}^2 \bar{\phi}_{l-n}^* \eta_{n-m}^1 - \\
& 16e^3 \sum_{m \neq 0, n} \bar{\eta}_{l-m}^1 \bar{\phi}_{l-n} \eta_{n-m}^2 + 16e^2 \sum_{n \neq 0} L^* \bar{\eta}_{l-n}^2 \eta_{l-n}^1 \\
& + 16e^2 \sum_{n \neq 0} L \bar{\eta}_{l-n}^1 \eta_{l-n}^2 + 4e^2 (\bar{\eta}_l^1 \partial \eta_l^1 + \bar{\eta}_l^2 \partial \eta_l^2) - \\
& - 4e^3 \sum_n (\bar{\eta}_n^1 \bar{A}_{n-l} \eta_l^1 + \bar{\eta}_n^2 \bar{A}_{n-l} \eta_l^2) + \\
& 12me^2 (\bar{\eta}_l^1 \eta_l^1 - \bar{\eta}_l^2 \eta_l^2) + 8e^3 \sum_n \bar{\eta}_n^2 \bar{\phi}_{n-l}^* \eta_l^1 - \\
& - 8e^3 \sum_n \bar{\eta}_n^1 \bar{\phi}_{n-l} \eta_l^2 + 8e^2 L^* \bar{\eta}_l^2 \eta_l^1 + 8e^2 L \bar{\eta}_l^1 \eta_l^2
\end{aligned}$$

Log divergences (coming from $a_4^{(d=6)}$).

$$\begin{aligned}
\Delta S_{\log} \equiv & \int \frac{d^4x}{(4\pi)^3} e^2 \sum_n \left(4 (\bar{\eta}_n^1 \partial \eta_n^1 + \right. \\
& \bar{\eta}_n^2 \partial \eta_n^2 + N \bar{\eta}_n^1 \eta_n^2 + N^* \bar{\eta}_n^2 \eta_n^1) \\
& + 12m (\bar{\eta}_n^1 \eta_n^1 - \bar{\eta}_n^2 \eta_n^2) + \frac{4}{3} (\bar{F}_{-n}^{\mu\nu} \bar{F}_{\mu\nu}^n + \\
& 2|N|^2 \bar{A}_{-n}^\mu \bar{A}_\mu^n - 4i \partial_\mu \bar{A}_{-n}^\mu \left(\frac{n_5}{R_5} \bar{A}_5^n + \frac{n_6}{R_6} \bar{A}_6^n \right) \\
& + 2\bar{A}_5^{-n} \left(-\square + \frac{n_6^2}{R_6^2} \right) \bar{A}_5^n + 2\bar{A}_6^{-n} \left(-\square + \frac{n_5^2}{R_5^2} \right) \bar{A}_6^n \\
& \left. - 4 \frac{n_5 n_6}{R_5 R_6} \bar{A}_5^{-n} \bar{A}_6^n \right) - \\
& - 4e \sum_m \left(\bar{\eta}_m^1 \bar{A}_{m-n} \eta_n^1 + \bar{\eta}_m^2 \bar{A}_{m-n} \eta_n^2 + \bar{\eta}_m^1 \bar{\phi}_{m-n} \eta_n^2 - \right. \\
& \left. \bar{\eta}_m^2 \bar{\phi}_{m-n}^* \eta_n^1 \right) \log \frac{\Lambda_{d=4}^2}{\mu_{d=4}^2}
\end{aligned}$$

The zero mode of the Log divergences .

(coming from $a_4^{(d=6)}$)

$$\begin{aligned} \Delta S_{\log} \equiv & \int \frac{d^4x}{(4\pi)^3} e^2 \left(4 (\bar{\eta}^1 \not{\partial} \eta^1 + \bar{\eta}^2 \not{\partial} \eta^2) \right. \\ & + 12m (\bar{\eta}^1 \eta^1 - \bar{\eta}^2 \eta^2) + \frac{4}{3} (\bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \\ & \left. - 2\bar{A}_5 \square \bar{A}_5 - 2\bar{A}_6 \square \bar{A}_6) - \right. \\ & \left. - 4e (\bar{\eta}^1 \bar{A} \eta^1 + \bar{\eta}^2 \bar{A} \eta^2 + \bar{\eta}^1 \bar{\phi} \eta^2 - \bar{\eta}^2 \bar{\phi}^* \eta^1) \right) \\ & \log \frac{\Lambda_{d=4}^2}{\mu_{d=4}^2} \end{aligned}$$

Now the scalar $\bar{\phi}$ is protected by the 6D WI

Log-log divergences (coming from $a_6^{(d=6)}$).

$$\begin{aligned}
\Delta S_{\log \log} \equiv & \int \frac{d^4 x}{(4\pi)^3} \frac{e^2}{M^2} \sum_n \left(-m\bar{\eta}_n^1 \left(\frac{19}{15} (-\square + \right. \right. \\
& |N|^2) + 2m\partial + 6m^2) \eta_n^1 + \\
& + m\bar{\eta}_n^2 \left(\frac{19}{15} (-\square + |N|^2) - 2m\partial + 6m^2 \right) \eta_n^2 - \\
& 2m^2 (N\bar{\eta}_n^1 \eta_n^2 + N^* \bar{\eta}_n^2 \eta_n^1) + \\
& + \frac{23}{9} \partial_\mu \bar{F}_{-n}^{\mu\nu} (\partial^\rho \bar{F}_{\rho\nu}^n - 2|N|^2 \bar{A}_\nu^n) - \\
& |N|^2) + \frac{4}{3} m^2) \bar{F}_n^{\mu\nu} + \\
& + |N|^2 \bar{A}_{-n}^\mu \left(\frac{31}{15} (-\square + |N|^2) - \frac{8}{3} m^2 \right) \bar{A}_\mu^n - \\
& - i\partial_\mu \bar{A}_{-n}^\mu \left(\frac{62}{15} (-\square + |N|^2) - \frac{16}{3} m^2 \right) \left(\frac{n_5}{R_5} \bar{A}_5^n + \right. \\
& \left. \frac{n_6}{R_6} \bar{A}_6^n \right) + \\
& + \bar{A}_5^{-n} \left(\frac{31}{15} (-\square + |N|^2) - \frac{8}{3} m^2 \right) \left(-\square + \frac{n_6^2}{R_6^2} \right) \bar{A}_5^n
\end{aligned}$$

$$\begin{aligned}
& + \bar{A}_6^{-n} \left(\frac{31}{15} (-\square + |N|^2) - \frac{8}{3} m^2 \right) \left(-\square + \frac{n_5^2}{R_5^2} \right) \bar{A}_6^n \\
& - \frac{n_5 n_6}{R_5 R_6} \bar{A}_5^{-n} \left(\frac{62}{15} (-\square + |N|^2) - \frac{16}{3} m^2 \right) \bar{A}_6^n \\
& \log \left(\frac{M^2}{\mu_{(d=6)}^2} \log \frac{\Lambda_{(d=4)}^2}{\mu_{(d=4)}^2} \right) + O(e^3)
\end{aligned}$$

Those are new non-standard counterterms
(albeit suppressed by inverse powers of M)

Consider now the reduction
of a renormalizable theory.

QED_4 in $\mathbb{R}^2 \times T^2$.

The standard logarithmic divergence in 4D

$$a_4 = \int \frac{d^4x}{(4\pi)^2} \left(\frac{2}{3} e^2 \bar{F}_{\mu\nu}^2 + 2e^2 \bar{\eta} \gamma^\mu \bar{D}_\mu \eta + 8e^2 m \bar{\eta} \eta \right)$$

Harmonic expansion.

$$\begin{aligned}
 S = \int d^2x \sum_{n_3, n_4} & \left(\bar{\psi}_n^1 \partial \psi_n^1 + \bar{\psi}_n^2 \partial \psi_n^2 + \right. \\
 & \bar{\psi}_n^1 \left(i \frac{n_3}{R_3} + \frac{n_4}{R_4} \right) \psi_n^2 - \bar{\psi}_n^2 \left(i \frac{n_3}{R_3} - \frac{n_4}{R_4} \right) \psi_n^1 + \\
 & \left. + m \left(\bar{\psi}_n^1 \psi_n^1 - \bar{\psi}_n^2 \psi_n^2 \right) \right. \\
 & - \frac{1}{2} (A_a^n)^* \left(\square - \frac{n_3^2}{R_3^2} - \frac{n_4^2}{R_4^2} \right) A_n^a - \\
 & \frac{1}{2} (A_3^n)^* \left(\square - \frac{n_3^2}{R_3^2} - \frac{n_4^2}{R_4^2} \right) A_3^n - \\
 & \left. - \frac{1}{2} (A_4^n)^* \left(\square - \frac{n_3^2}{R_3^2} - \frac{n_4^2}{R_4^2} \right) A_4^n - \right. \\
 & e \sum_m \left(\bar{\psi}_m^1 A_{m-n} \psi_n^1 + \bar{\psi}_m^2 A_{m-n} \psi_n^2 + \right. \\
 & \left. + \bar{\psi}_m^1 A_3^{m-n} \psi_n^2 - \bar{\psi}_m^2 A_3^{m-n} \psi_n^1 - \right. \\
 & \left. \left. i \bar{\psi}_m^1 A_4^{m-n} \psi_n^2 - i \bar{\psi}_m^2 A_4^{m-n} \psi_n^1 \right) \right)
 \end{aligned}$$

Zero mode

$$S = \int d^2x \left(\bar{\psi}^1 \not{\partial} \psi^1 + \bar{\psi}^2 \not{\partial} \psi^2 + m (\bar{\psi}^1 \psi^1 - \bar{\psi}^2 \psi^2) \right. \\ \left. - \frac{1}{2} A_a \square A^a - \frac{1}{2} \phi^* \square \phi \right. \\ \left. - e (\bar{\psi}^1 \not{A} \psi^1 + \bar{\psi}^2 \not{A} \psi^2 + \bar{\psi}^1 \phi \psi^2 - \bar{\psi}^2 \phi^* \psi^1) \right)$$

Two-dimensional tower

$$\Delta S_{tower} = \frac{1}{\epsilon} a_2 = \frac{1}{\epsilon} \int \frac{d^2x}{4\pi} \sum_l 4 (m^2 - |L|^2 - e^2 \sum_n \bar{\phi}_n^* \bar{\phi}_{-n} + e (L^* \bar{\phi}_0 - L \bar{\phi}_0^*))$$

The scalar is not protected by the WI

True counterterm

$$\begin{aligned}
 a_4 = & \int \frac{d^2x}{(4\pi)^2} \frac{e^2}{M^2} \sum_n \left(2 (\bar{\eta}_n^1 \partial \eta_n^1 + \bar{\eta}_n^2 \partial \eta_n^2 + \right. \\
 & N \bar{\eta}_n^1 \eta_n^2 + N^* \bar{\eta}_n^2 \eta_n^1) + 8m (\bar{\eta}_n^1 \eta_n^1 - \bar{\eta}_n^2 \eta_n^2) \\
 & + \frac{2}{3} (\bar{F}_{-n}^{ab} \bar{F}_{ab}^n + 2|N|^2 \bar{A}_{-n}^a \bar{A}_a^n - \\
 & 4i \partial_a \bar{A}_{-n}^a \left(\frac{n_3}{R_3} \bar{A}_3^n + \frac{n_4}{R_4} \bar{A}_4^n + \right) + \\
 & 2\bar{A}_3^{-n} \left(-\square + \frac{n_4^2}{R_4^2} \right) \bar{A}_3^n + 2\bar{A}_4^{-n} \left(-\square + \frac{n_3^2}{R_3^2} \right) \bar{A}_4^n - \\
 & \left. 4 \frac{n_3 n_4}{R_3 R_4} \bar{A}_3^{-n} \bar{A}_4^n \right) - \\
 & -2e \sum_m \left(\bar{\eta}_m^1 \bar{A}_{m-n} \eta_n^1 + \bar{\eta}_m^2 \bar{A}_{m-n} \eta_n^2 + \right. \\
 & \left. \bar{\eta}_m^1 \bar{\phi}_{m-n} \eta_n^2 - \bar{\eta}_m^2 \bar{\phi}_{m-n}^* \eta_n^1 \right)
 \end{aligned}$$

Zero mode of the expansion of the true counterterm

$$a_4^{(0)} = \int \frac{d^2x}{(4\pi)^2} \frac{e^2}{M^2} \left(2 (\bar{\eta}^1 \not{\partial} \eta^1 + \bar{\eta}^2 \not{\partial} \eta^2) + \right. \\ \left. 8m (\bar{\eta}^1 \eta^1 - \bar{\eta}^2 \eta^2) + \right. \\ \left. \frac{2}{3} \bar{F}^{ab} F_{ab} - \frac{4}{3} \bar{\phi}^* \square \bar{\phi} - \right. \\ \left. -2e (\bar{\eta}^1 \bar{A} \eta^1 + \bar{\eta}^2 \bar{A} \eta^2 + \bar{\eta}^1 \bar{\phi} \eta^2 - \bar{\eta}^2 \bar{\phi}^* \eta^1) \right)$$

Again, the 2D scalar is now protected.

CONCLUSIONS

- **This is the correct way to implement the physical intuition**
- **Physics at very short distances is truly higher dimensional.**
- **Nonstandard counterterms are needed**