Gauge Symmetries for Spin-2 Theories

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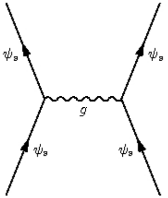
Outline

Massless Spin-2 and Gravitation

We expect *gravity* to be mediated by a massless field g of a given spin.



1. Fermions are ruled out ($\Delta J = \pm 1/2$ or wrong radial dependence)
2. Spin 0 boson does not deviate light (couples to the trace)
3. Spin $2n-1$, $\forall n \in \mathbb{Z}$ alike charges repel.
4. Linear Spin 2 wrong Mercury perihelion (corrected by non-linearities).

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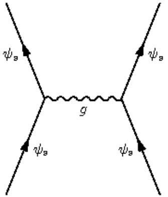

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TDiff Gauge Symmetry and Massless Spin-2

- Massless particle of spin-2: 2 polarizations.
- To describe them with a tensor $h_{\mu\nu} = 2 \oplus 1 \oplus 0 \oplus 0$, a gauge symmetry is required.
- The minimal is TRANSVERSE (linear) Diff.

$$h_{\mu\nu} \sim h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}, \quad \partial^\mu \xi_\mu = 0$$

van der Bij, van Dam & Ng, 81

They leave $h = \eta^{\mu\nu} h_{\mu\nu}$ invariant.

TDiff Lagrangians from Consistency

The most general covariant massless lagrangian for $h_{\mu\nu}$ is

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\partial_\mu h^{\nu\rho}\partial^\mu h_{\nu\rho} - \frac{\beta}{2}\partial_\mu h^{\mu\rho}\partial_\nu h^\nu_\rho \\ &+ \frac{a}{2}\partial^\mu h \partial^\rho h_{\mu\rho} - \frac{b}{4}\partial_\mu h \partial^\mu h\end{aligned}$$

D.o.f

$$\begin{aligned} h_{00} &= 2A \\ h_{0i} &= \partial_i B + V_i \\ h_{ij} &= -2\psi\delta_{ij} + 2\partial_i\partial_j E + 2\partial_{(i}F_{j)} + t_{ij} \end{aligned}$$

with $\partial^i F_i = \partial^i V_i = \partial^i t_{ij} = t^i_i = 0$.

$\beta = 1$ from ghost free vectors F_i, $V_i \Rightarrow$ TRANSVERSE Diff..

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Ghost and Tachyon Free TDiff Lagrangians

$$\mathcal{L}_m = \frac{1}{4}\partial_\mu h^{\nu\rho}\partial^\mu h_{\nu\rho} - \frac{1}{2}\partial_\mu h^{\mu\rho}\partial_\nu h^\nu_\rho + \frac{a}{2}\partial^\mu h \partial^\rho h_{\mu\rho} - \frac{b}{4}\partial_\mu h \partial^\mu h - \frac{1}{2}m^2 h^2$$

ψ : Massive Spin-0 of mass m.

t_{ij} : Massless Spin-2.

For $n \neq 2$,

$$b \leq \frac{1-2a+(n-1)a^2}{(n-2)}, \quad m^2 > 0.$$

- Phenomenology: standard scalar-tensor ($m \gtrsim (30\mu m)^{-1}$) Will, 05
- Pure spin-2

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van Nieuwenhuizen, 73

Pure Spin-2 Lagrangians

Almost all come from usual Fierz-Pauli ($a = b = 1$) after field redef.
One pathological case:

Two INEQUIVALENT lagrangians which only propagate massless spin-2 (and enlarged Gauge Symmetry)!!:

- Fierz-Pauli (GR): $a = b = 1$ (and field redefinitions).
  G. Symmetry: Diff

  $$\delta h_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)}.$$

  Only possibility for massive spin-2.

  van Nieuwenhuizen, 73

- WTDiff: $a = \frac{2}{n}$, $b = \frac{n+2}{n^2}$.
  G. Symmetry: Weyl & TDiff

  $$\delta h_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)},\ \partial^\mu \xi_\mu = 0, \quad \delta h_{\mu\nu} = 2\eta_{\mu\nu}\phi.$$

TDiff Non-Linear Extension

From the linear symmetries:

- $f(\eta_{\mu\nu}, g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}) = h + O(h^2)$ as an invariant.

$$S[g_{\mu\nu}, \psi, \eta_{\mu\nu}] = \int \mathrm{d}^n x L(f, g_{\mu\nu}, \psi)$$

TDiff Non-Linear Extension

From the linear symmetries:

- Most natural possibilty: $f = g \equiv \det g_{\mu\nu}$

$$S[g_{\mu\nu},\psi] = \int \mathrm{d}^n x \left(-\Phi(g,\psi) R(\hat{g}_{\mu\nu}) + L[g,\psi,\hat{g}_{\mu\nu}]\right)$$

$\hat{g}_{\mu\nu} = |g|^{-1/n} g_{\mu\nu}$.

- Invariant under Diff. of $J = \det \frac{\partial x}{\partial y} = 1$. (TDiff)
- Constraint by the linear analysis.

E.o.m.: Scalar-Tensor.

Einstein's frame: $\bar{g}_{\mu\nu} = \Phi \hat{g}_{\mu\nu}$

Buchmuller & Dragon , 88

$$G_{\mu\nu}(\bar{g}_{\mu\nu}) = \kappa \bar{T}_{\mu\nu}(g,\psi) + \Lambda \bar{g}_{\mu\nu}.$$

- g as a scalar.
- Λ is an integration constant.

Non-Linear Extension: Pure GR

- Weyl Symmetry ($g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$). $\delta_W \hat{g}_{\mu\nu} = \delta_W g^{-1/n} g_{\mu\nu} = 0$.

$$S[g_{\mu\nu}, \psi] = \int \mathrm{d}^n x \left(-\hat{g}^{\mu\nu} R_{\mu\nu}(\hat{g}_{\mu\nu}) + L_W[g, \psi, \hat{g}_{\mu\nu}]\right)$$ $\sim$ Unruh , 89

E.o.m.

$$R_{\mu\nu} - \frac{1}{n} g_{\mu\nu} R = f_{\mu\nu}(g)$$

In the gauge $|g| = 1$, $f_{\mu\nu}(g) = 0$ thus using geometrical Bianchi:

$$2\nabla^\mu R_{\mu\nu} = \nabla_\nu R, \implies \nabla_\nu R = 0$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \Lambda g_{\mu\nu}$$

- $S = \int (\sqrt{-g} R + f(g)) \mathrm{d}^n x$ (SSB?, Anomalies?)
- Other $\eta_{\mu\nu}$ dependent possibilities.

Non-Linear Extension (Gupta's Program)

Hope: Finding a non-linear extension through self-interacion.

E.o.m. $\mathcal{D}(h)_{\mu\nu} = \frac{\delta \mathcal{L}_{(m=0)}}{\delta h^{\mu\nu}} = 0$

TDiff: $\partial^\mu \mathcal{D}(h)_{\mu\nu} = \partial_\nu \Phi$
WTDiff: $\partial^\mu \mathcal{D}(h)_{\mu\nu} = \partial_\nu \Phi, \ \eta^{\mu\nu}\mathcal{D}(h)_{\mu\nu} = 0.$

The natural equation to consider is $T_{\mu\nu}(h) + \beta T(h)\eta_{\mu\nu} = \mathcal{D}(h)_{\mu\nu}$.
But:

- Wald's (also Damour, Henneaux et alt.) extension not clear.
- Deser's method OK but not what we expect.

- It is not clear how to extend uniquely non-linearly $\partial_\mu \xi^\mu = 0$.
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Conclusions

- The minimal gauge symmetry for massless spin-2 is TDiff:

$$\delta h_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)}, \quad \partial^{\mu}\xi_{\mu} = 0$$

- Massless Spin-2 admits naturally a (well-behaved) scalar partner, except
  - Fierz-Pauli (Diff invariant).
  - WTDiff (Weyl & TDiff invariant).
- We found (not unique) non-linear completions equivalent to Scalar-Tensor. Restricting to GR: Diff and WTDiff have equivalent e.o.m. except for the origin of Λ.
- Quantization (BRST) is possible and the anomalies are similar to the usual gravitational anomalies. Still maybe quantum differences.