

Vacuum Energy and Renormalization on the Edge

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Quantum Fields and Boundary Conditions

Edges and boundaries:

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- **Defects and space singularities**
(Black holes, domain walls, cosmic strings, ...)

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- Topology change and Edge states
- Holographic principle, Topological Field Theories, D-branes and all that

Quantum Boundaries: **Scalar Fields**

- Physical space Ω metric g and boundary $\partial\Omega$
- Free scalar field

$$S_B(\phi) = \frac{1}{2} \int dt \int_{\Omega} \sqrt{g} d^{D+1}x \left[|\dot{\phi}|^2 - |\nabla\phi|^2 - V(|\phi|^2) \right]$$

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- In terms of boundary values of fields

$$\varphi = \phi|_{\partial\Omega} \quad \partial_n \varphi = \partial_n \phi|_{\partial\Omega}$$

Selfadjoint boundary conditions:

Ω orientable with regular boundary $\partial\Omega$

Balance defect of gradient term

$$\langle \phi_1, \Delta \phi_2 \rangle = \langle \Delta \phi_1, \phi_2 \rangle - \int_{\partial\Omega} \sqrt{g_{\partial\Omega}} d^D x \left[(\partial_n \phi_1^*) \phi_2 - \phi_1^* \partial_n \phi_2 \right]_{\partial\Omega}$$

Boundary flux term

$$\Sigma(\varphi_1, \varphi_2) = i \int_{\partial\Omega} \sqrt{g_{\partial\Omega}} d^D x \left[(\partial_n \varphi_1^*) \varphi_2 - \varphi_1^* \partial_n \varphi_2 \right]_{\partial\Omega}$$

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- The set \mathcal{M} of boundary conditions for **self-adjoint** extensions of Δ is in one-to-one correspondence with the group of **unitary** operators of $L^2(\partial\Omega, \mathbb{C}^N)$.

$$\varphi - i\partial_n \varphi = U(\varphi + i\partial_n \varphi)$$

Self-Adjoint Boundary Conditions

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- Group structure of the space of b.c.
- Global theory of boundary conditions
- Dynamical RG flow of boundary conditions

String theory

One-dimensional space $M = [0, \pi] \in \mathbb{R}$

1. Dirichlet boundary conditions

$$U = -\mathbb{I} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \varphi(0) = \varphi(\pi) = 0$$

2. Neumann boundary conditions

$$U = \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \varphi'(0) = \varphi'(\pi) = 0$$

3. Periodic boundary conditions

$$U = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \varphi(0) = \varphi(\pi)$$

$$M = \cup_{i=1}^N [a_i, b_i] \in \mathbb{R}$$

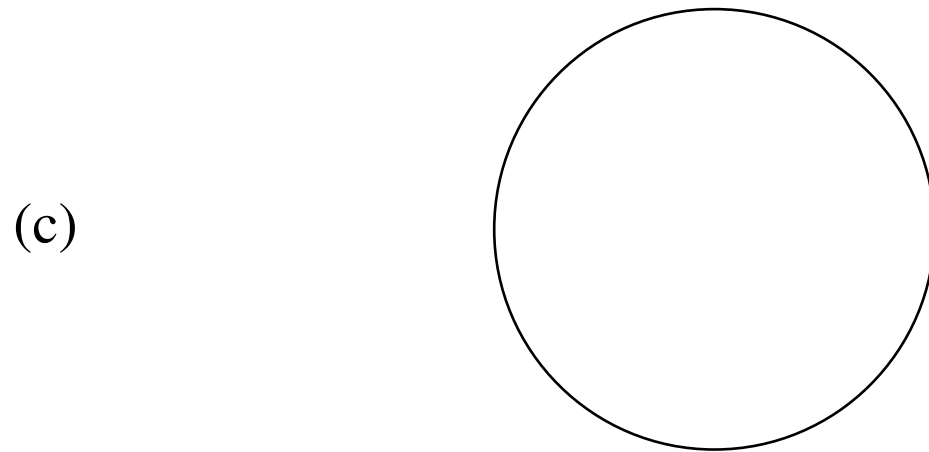
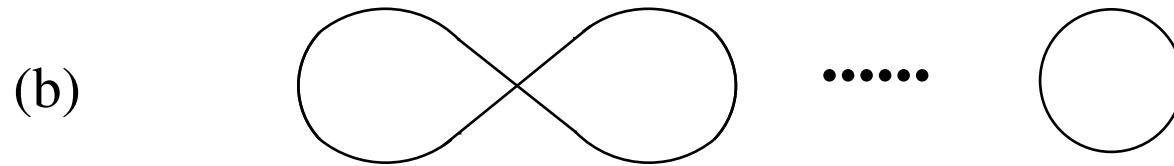
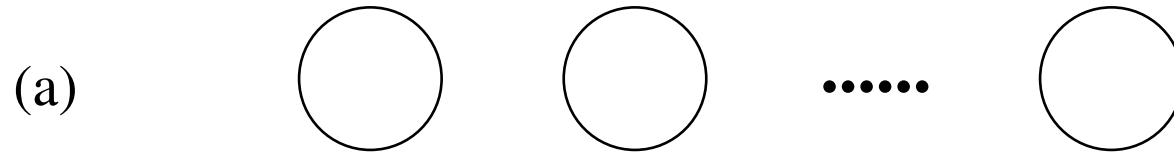
4. One single circle

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix}$$

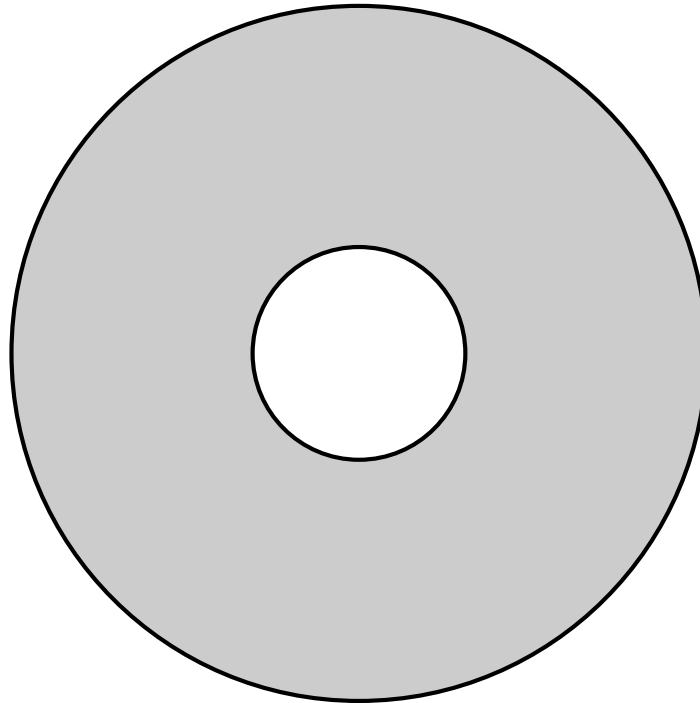
5. N Disconnected circles

$$U_N = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

TOPOLOGY CHANGE



Chiral boundary conditions and edge states



In the annulus there are chiral boundary conditions:

$$\partial_n \varphi_{out} = i \partial_\vartheta \varphi_{out},$$

$$\partial_n \varphi_{in} = -i \partial_\vartheta \varphi_{in}$$

with **chiral edges** localized at the boundaries

Cayley Transform

1. If $-1 \notin \text{Sp}U$ the boundary condition reduces to

$$\partial_n \varphi = -i \frac{\mathbb{I} - U}{\mathbb{I} + U} \varphi = A \varphi$$

2. If $1 \notin \text{Sp}U$ the boundary condition reduces to

$$\varphi = i \frac{\mathbb{I} + U}{\mathbb{I} - U} \partial_n \varphi = A^{-1} \partial_n \varphi$$

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Cayley transform

$$A = -i \frac{\mathbb{I} - U}{\mathbb{I} + U}$$

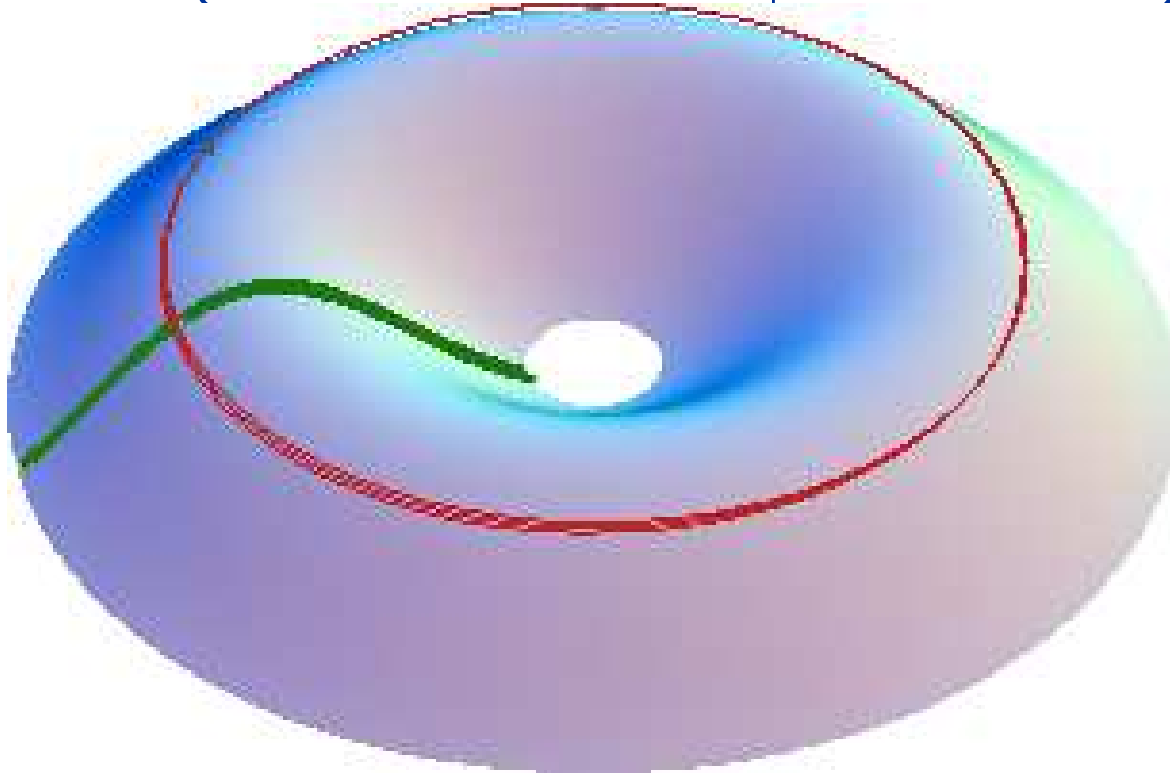
Inverse Cayley transform

$$U = \frac{\mathbb{I} - iA}{\mathbb{I} + iA}$$

Cayley submanifolds

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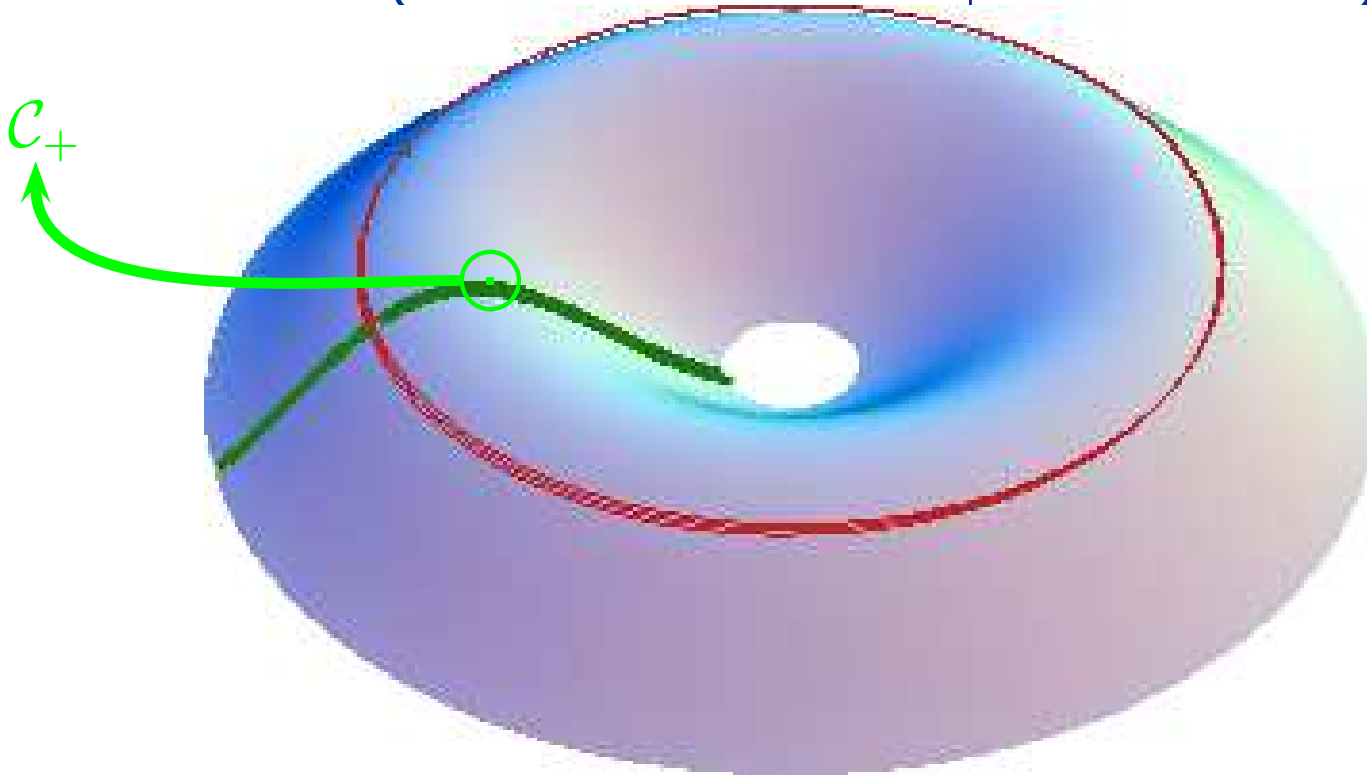
$$\mathcal{C}_{\pm} = \left\{ U \in \mathcal{U}(L^2(\Gamma, \mathbb{C}^N)) \mid \pm 1 \in \text{Sp}(U) \right\}$$



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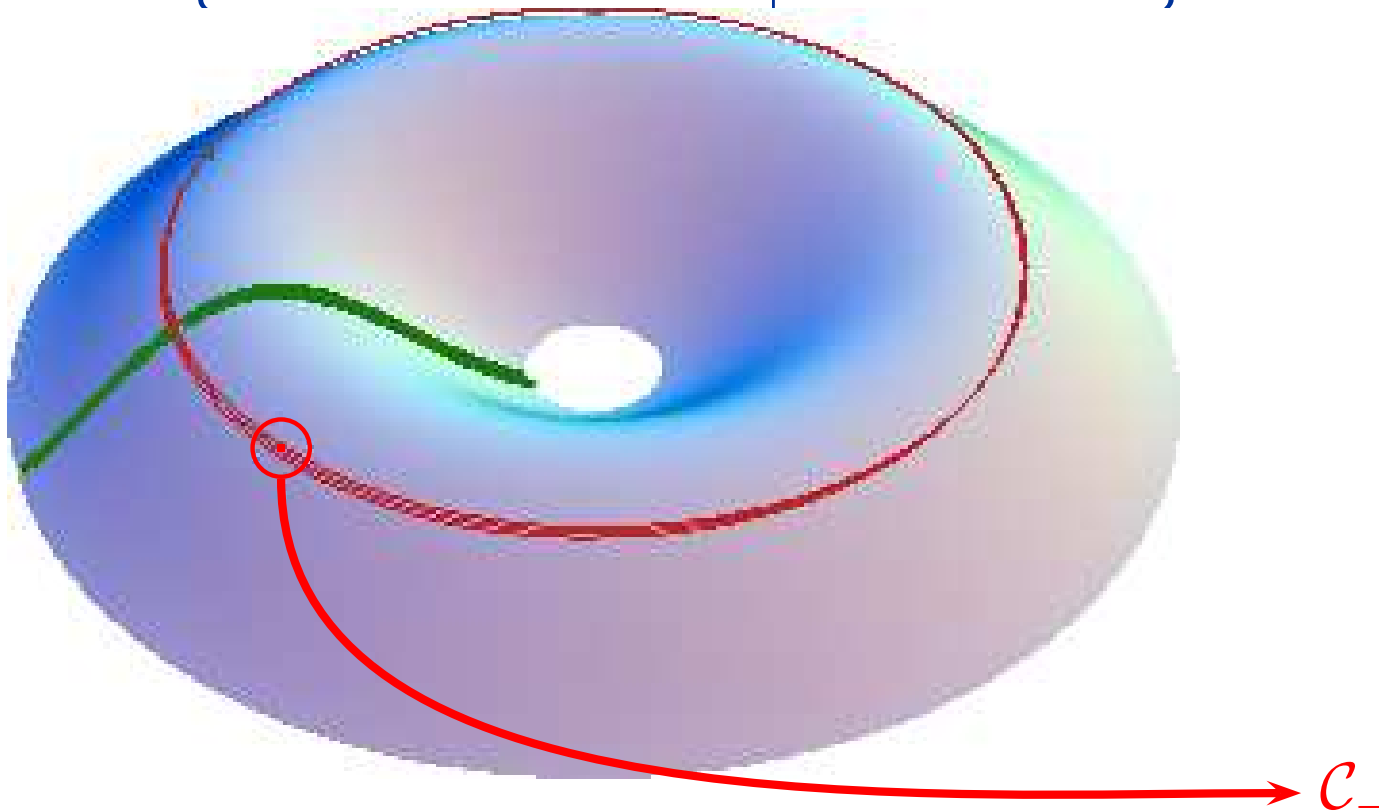
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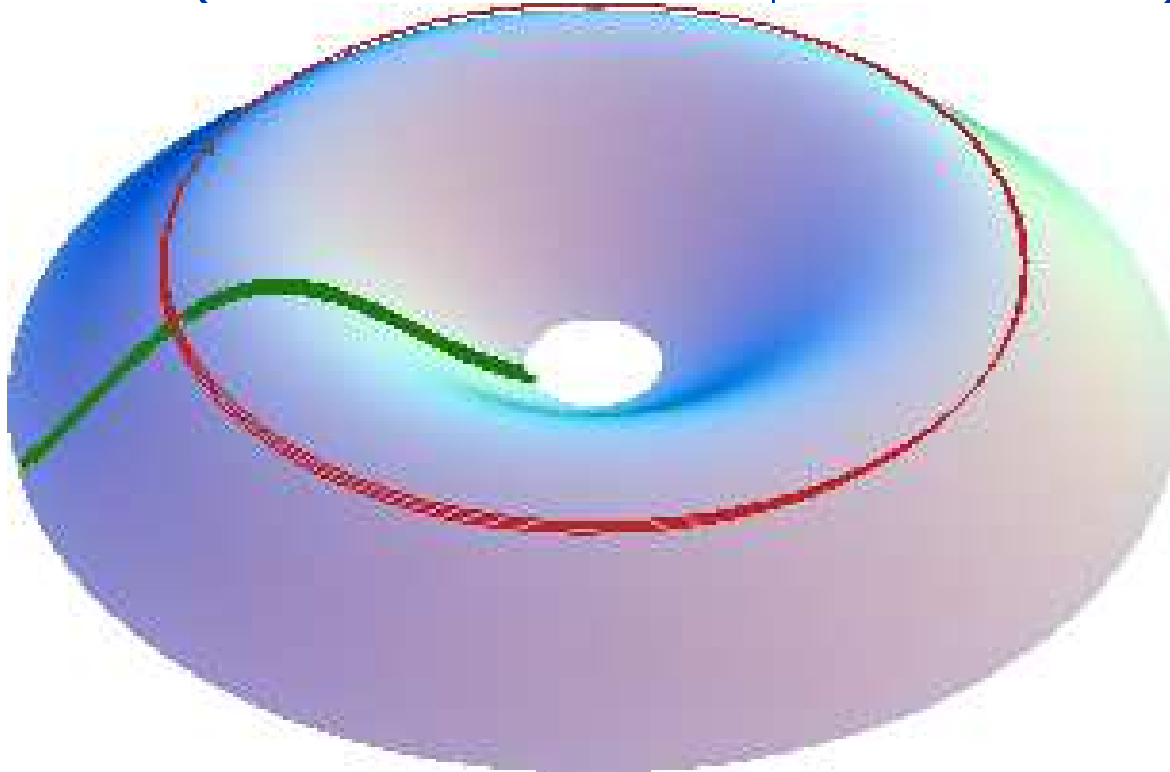
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The topology of the space of selfadjoint boundary condition is non-trivial

$$\pi_1 [\mathcal{U}(L^2(\partial\Omega, \mathbb{C}^N))] = \mathbb{Z}$$

Cayley submanifolds. Maslov index

The Maslov index of a closed path

$\gamma: S^1 \rightarrow \mathcal{U}(L^2(\partial\Omega, \mathbb{C}^N))$ of selfadjoint boundary conditions is

$$\nu_M(\gamma) = \frac{1}{2\pi} \int_0^{2\pi} \partial_{\vartheta} \log \det'(\gamma(\vartheta)) d\vartheta$$

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- The Maslov index of a closed path γ is equal to the indexed sum of crossing of γ through the Cayley submanifold \mathcal{C}_-

$$\nu_M(\gamma) = \int_0^{2\pi} \partial_\vartheta n(\gamma(\vartheta)) d\vartheta$$

Topology change and edge states

Now Δ with arbitrary boundary condition may not be a positive operator:

$$(\Psi_1, \Delta\Psi_2) = (\nabla\Psi_1, \nabla\Psi_2) - (\varphi_1, A\varphi_2)$$

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- In field theory only boundary conditions with $\Delta^U > 0$ positive are admissible by unitarity
- For any selfadjoint extension Δ^U of Δ with $-1 \in \text{Sp}U$ and smooth eigenfunction, the family of selfadjoint extensions Δ^{U_t} with $U_t = Ue^{it}$ and $0 < t \ll 2\pi$ has one negative energy level E_- which corresponds to an **edge state**: $E_- \rightarrow -\infty$ as $t \rightarrow 0$

Boundary Conditions and Symmetries

Real Scalar Fields

CP symmetry $U^\dagger = U^*$, $U = U^T$

$U = \pm \mathbb{I}$ (Neumann, Dirichlet)

$$U = \begin{pmatrix} A_1 & B \\ B^T & A_2 \end{pmatrix}$$

$$A_1 = A_1^T, A_2 = A_2^T, A_1 B^* + B A_2^\dagger = 0$$

$$B B^\dagger + A_1 A_1^\dagger = \mathbb{I}, A_2 A_2^\dagger + B^T B^* = \mathbb{I}$$

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Example: $\partial_n \varphi(L) = -M \partial_n \varphi(0)$; $\varphi(L) = M^{-1} \varphi(0)$;
 $M = M^\dagger = M^*$ (quasi-periodic)

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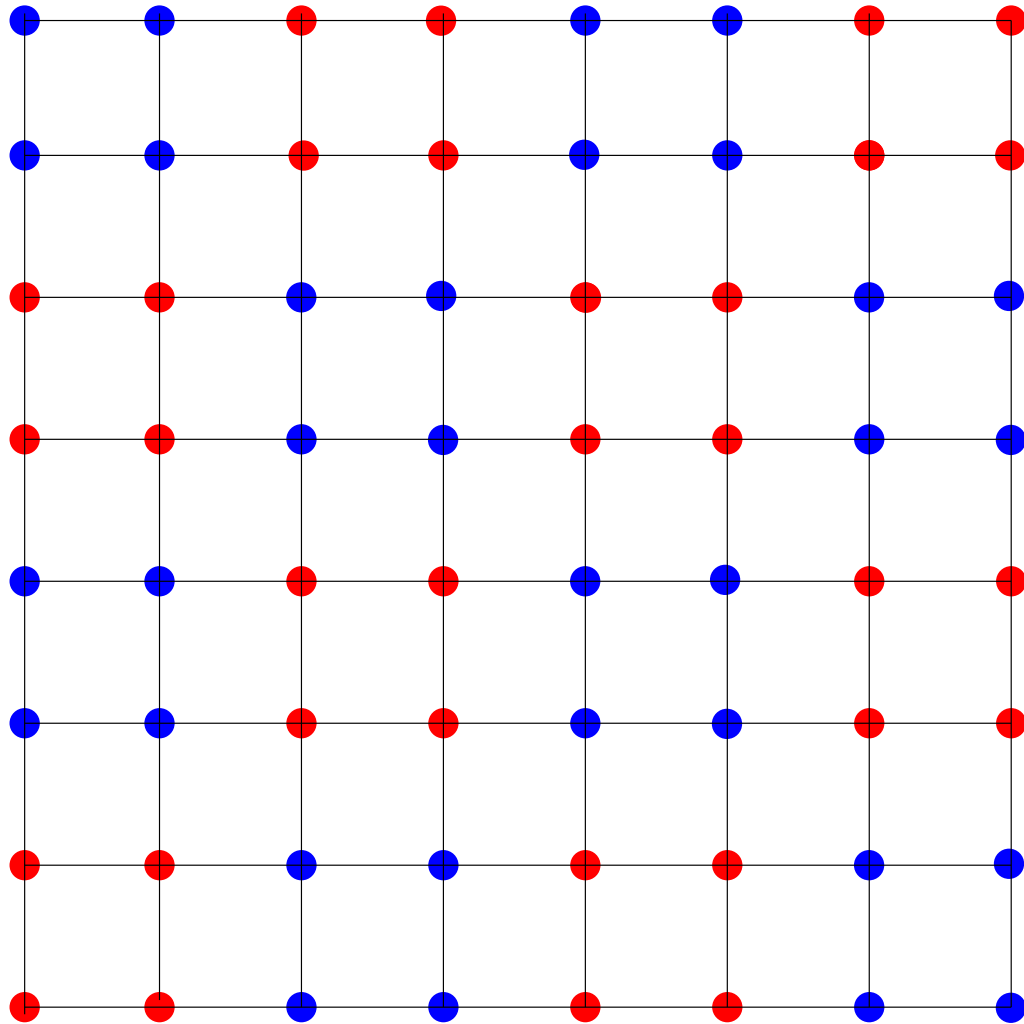
One Real Scalar Field

$$U = a_0 \mathbb{I} + i \vec{a} \cdot \vec{\sigma}$$

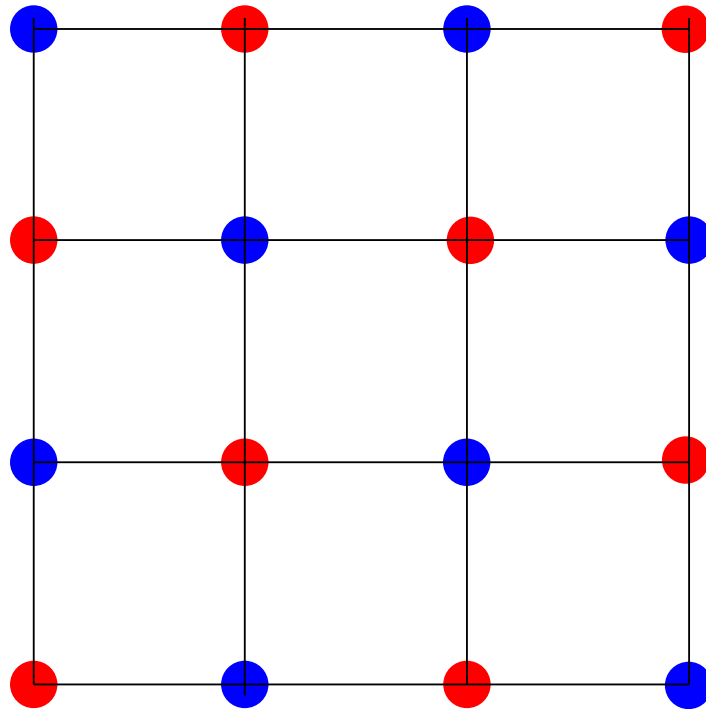
$$U = \pm \mathbb{I} \text{ (Neumann, Dirichlet)}$$

$$U = \cos \alpha \sigma_x + \sin \alpha \sigma_z \text{ (quasi - periodic)}$$

RG and Boundary Conditions



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RG and Boundary Conditions

In the continuum RG is defined by

$$\phi_L \left(\frac{x}{L} \right) = L^{[\phi]} [\phi(x) - \xi_L(x)]$$

$\xi_L \in \mathbb{C}_0^\infty(\Omega)$ is short range ($\frac{1}{L}$) fluctuating field

$$\partial_n \phi_L \left(\frac{x_b}{L} \right) = L^{[\phi]+1} [\partial_n \phi(x_b) - \partial_n \xi_L(x_b)] = A L^{[\phi]+1} \phi(x_b) = A_L \phi_L \left(\frac{x_b}{L} \right)$$

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The boundary condition

$$\partial_n \varphi = A \varphi$$

becomes renormalized

$$\partial_n \varphi_L = A_L \varphi_L$$

with $A_L = L^{[\phi]+1} A$.

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For more general boundary conditions the continuum RG is defined by

$$\Lambda U_L^\dagger \partial_L U_L = \frac{1}{2} (U_L^\dagger - U_L)$$

or

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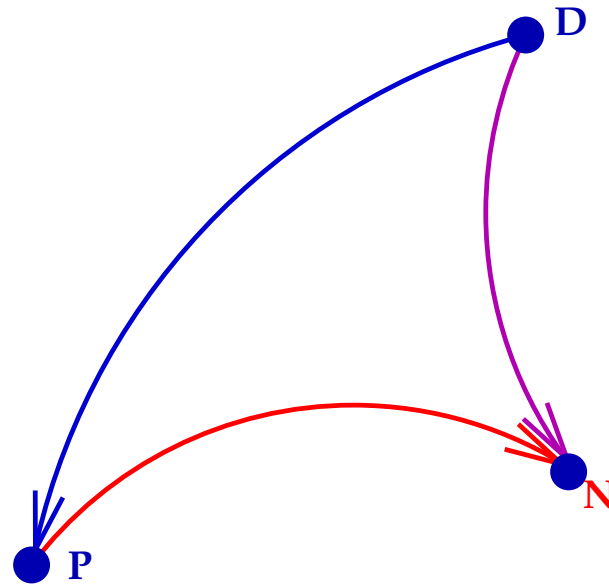
for $L = L_0 e^t$.

- Fixed points correspond to selfadjoint bc $U^\dagger = U$
- Dirichlet and Neumann bc ($U = \mp \mathbb{I}$) are **RG invariant**
- For mixed boundary conditions the RG flow goes from Dirichlet (**UV**) to Neumann (**IR**)

$$U = e^{2i \arctan e^{-t}} \mathbb{I}$$

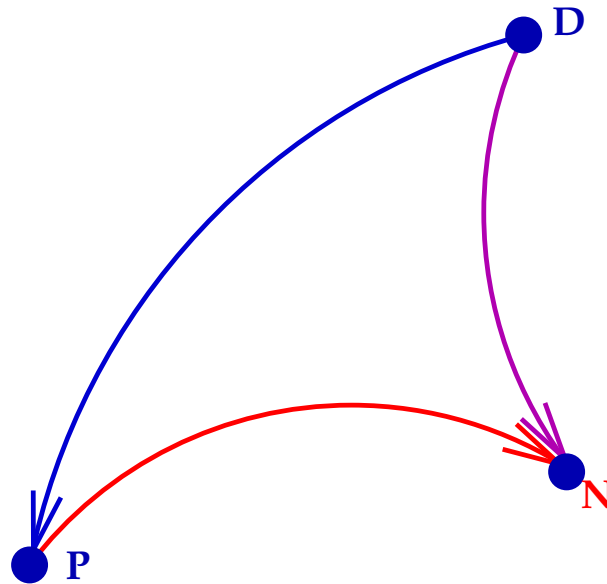
Cyclic RG and Boundary Conditions

- Critical exponents of boundary conditions RG flow are ± 1

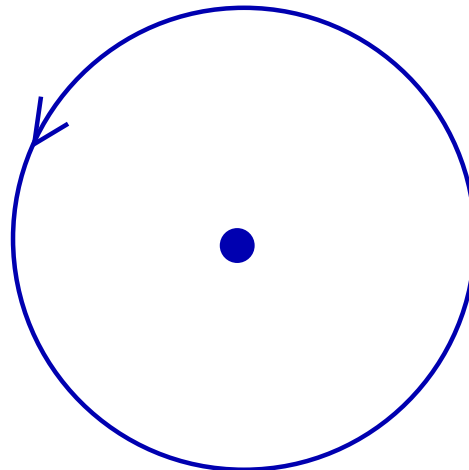


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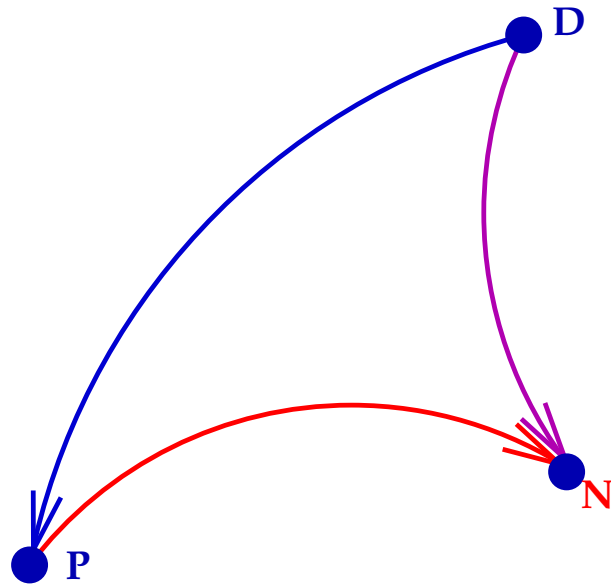


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Cyclic RG and Boundary Conditions

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- Regular boundaries do not have cyclic RG flows
- Cyclic RG with circle limits appear in some quantum systems with singular boundaries. [Glazek-Wilson]
- Also in some topological field theories as Russian Doll model of superconductivity [LeClair-Sierra]

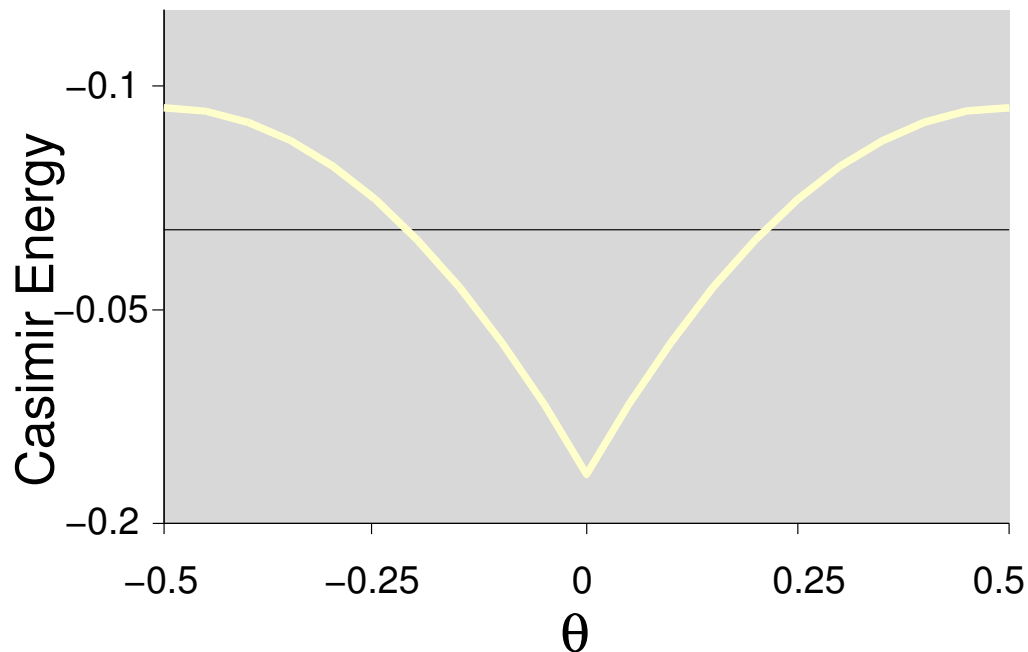
Casimir Energy: Pseudo-periodic b.c.

$$U = \cos 2\pi\vartheta \sigma_x - \sin 2\pi\vartheta \sigma_y$$

Vacuum Energy

$$E_0 = \frac{\pi}{L} \left(\frac{1}{12} - \left(|\vartheta| - \frac{1}{2} \right)^2 \right)$$

Pseudo-periodic boundary Conditions

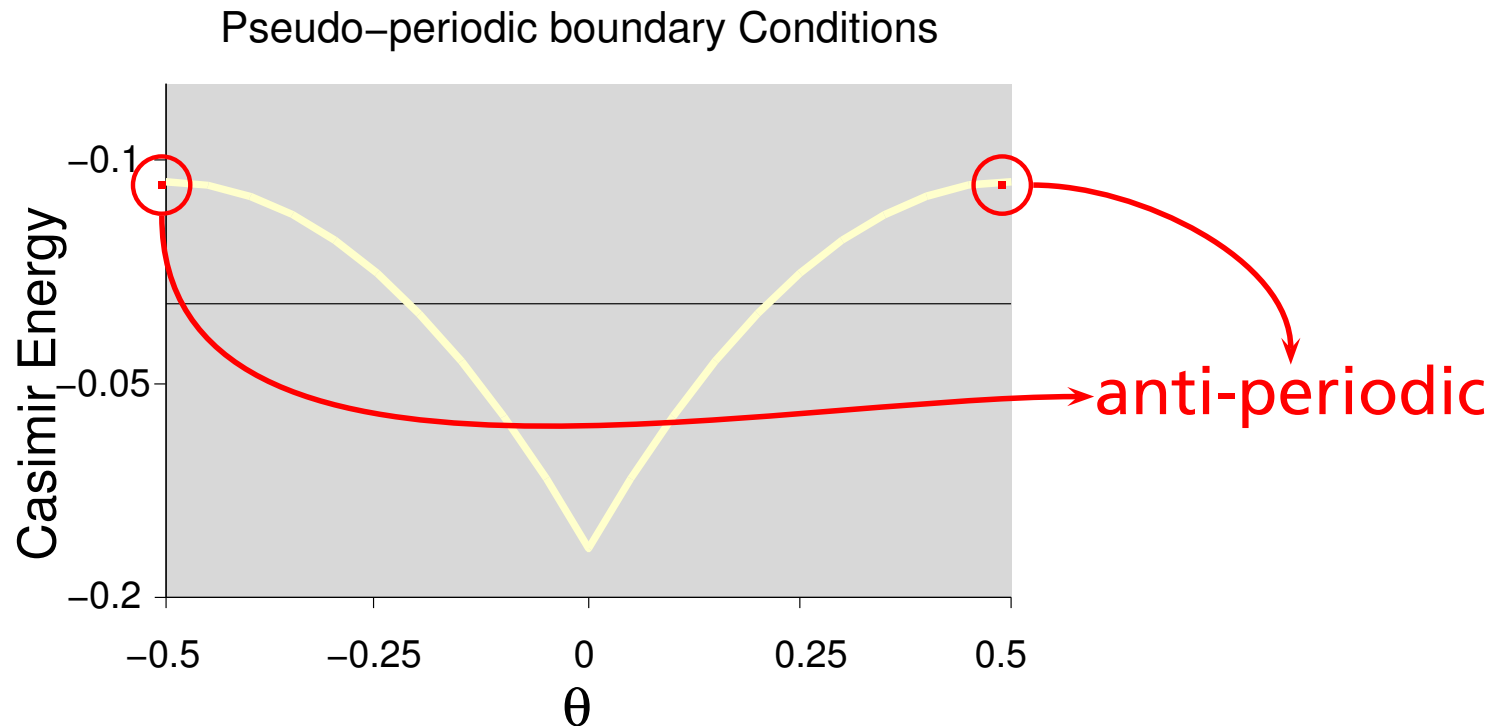


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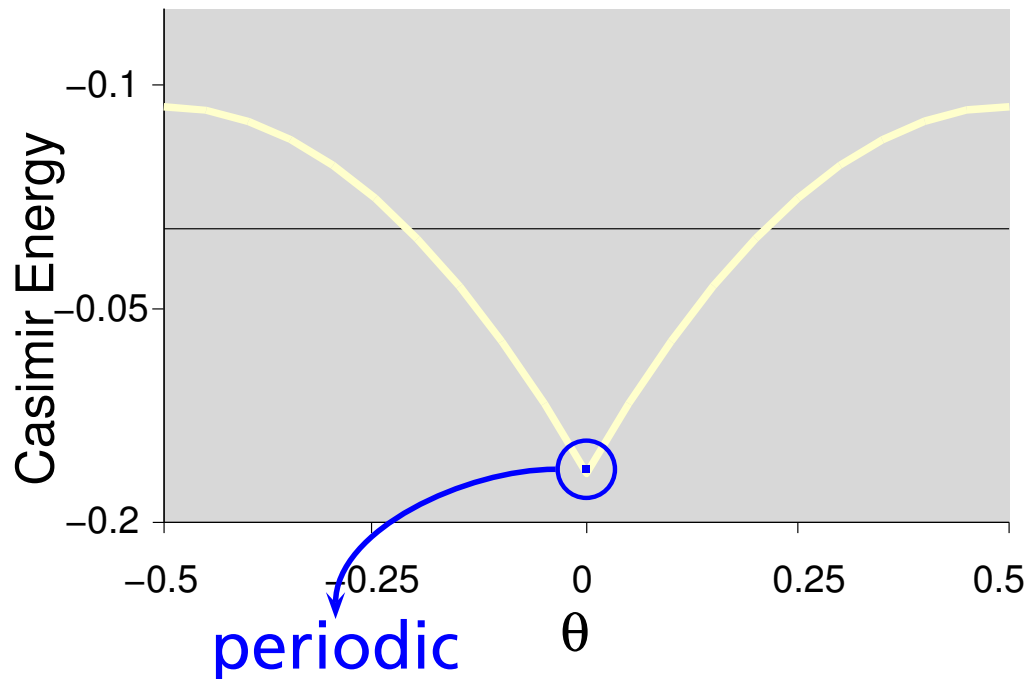
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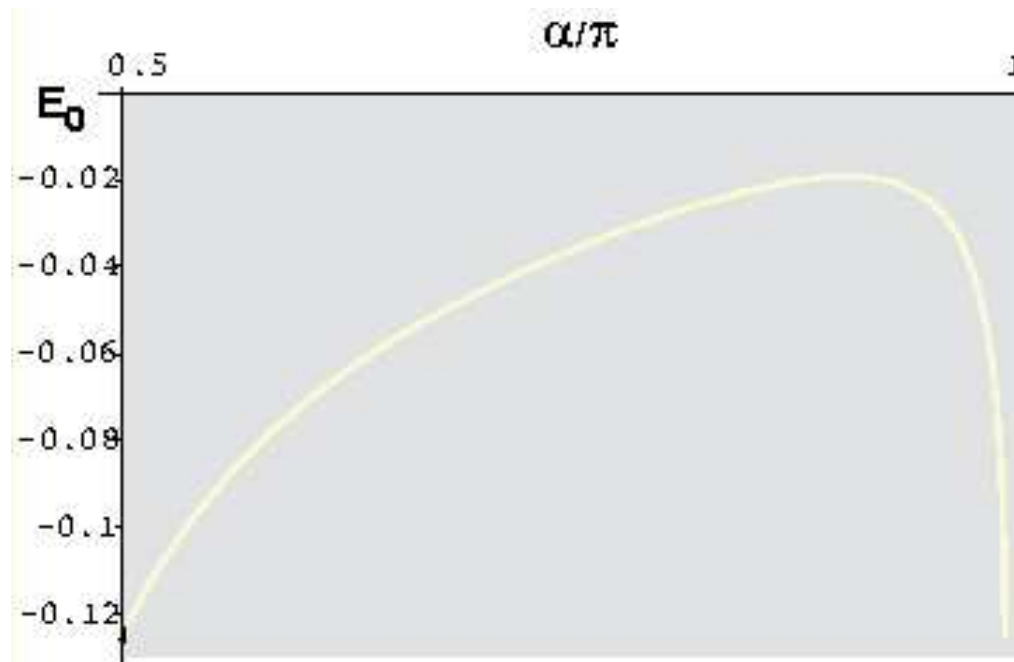


Casimir Vacuum Energy

Robin b.c.: $U = e^{i\alpha} \mathbb{I}$

$$\dot{\varphi}(0) = \frac{\tan \alpha}{L} \varphi(0), \quad \dot{\varphi}(L) = \frac{\tan \alpha}{L} \varphi(L)$$

Vacuum Energy

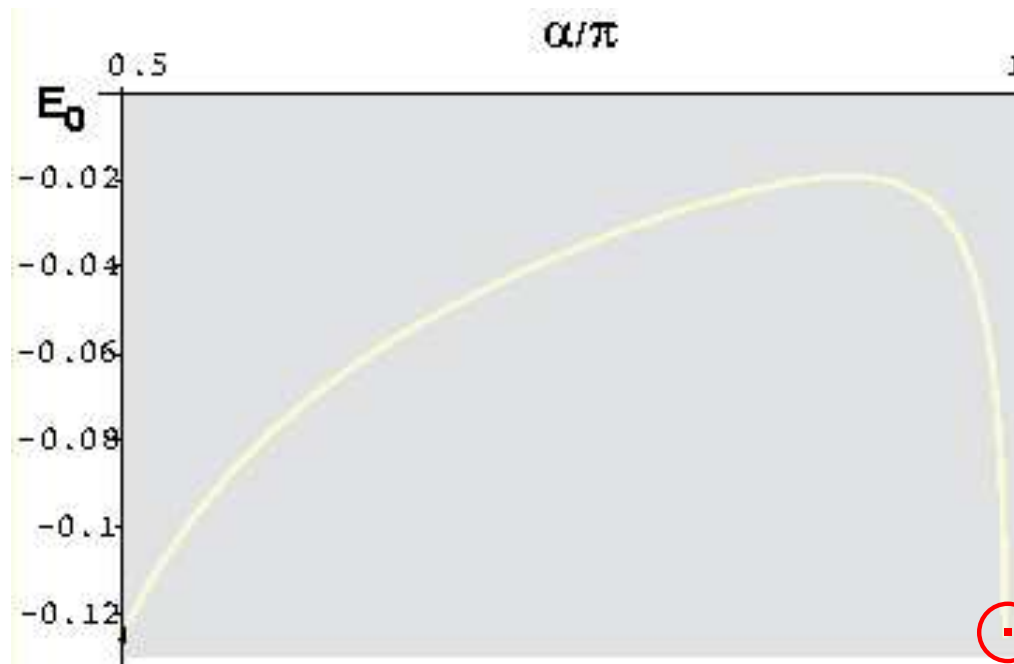


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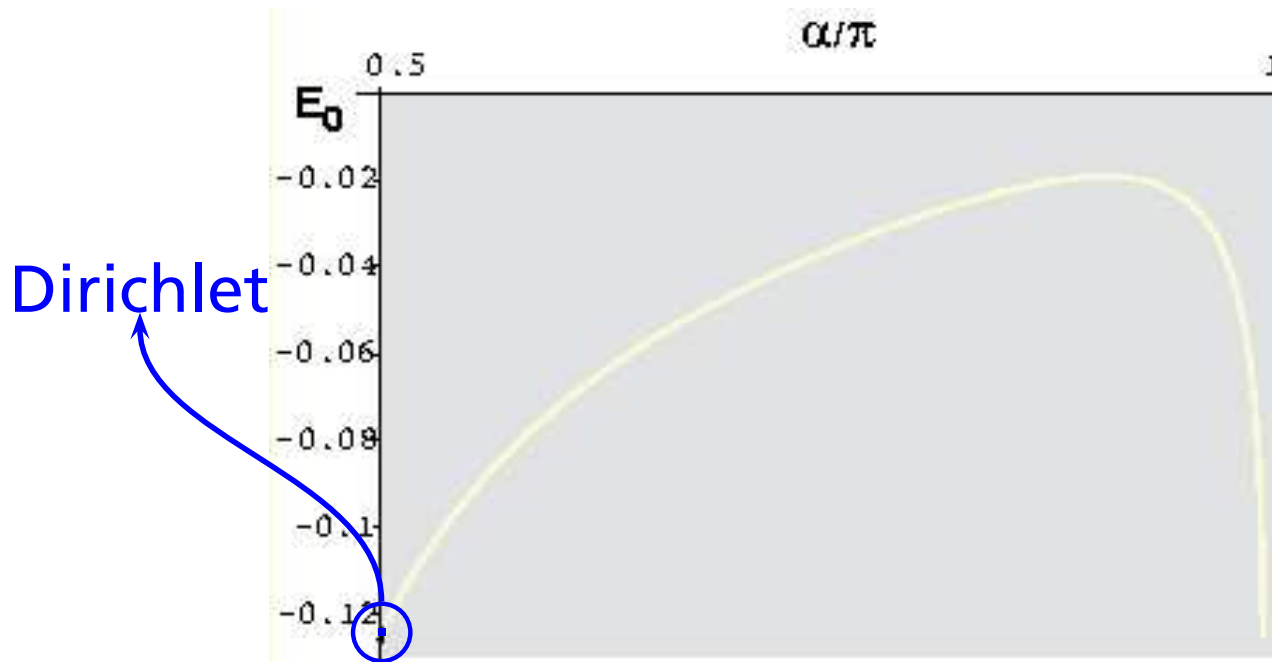
Neumann

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Vacuum Energy



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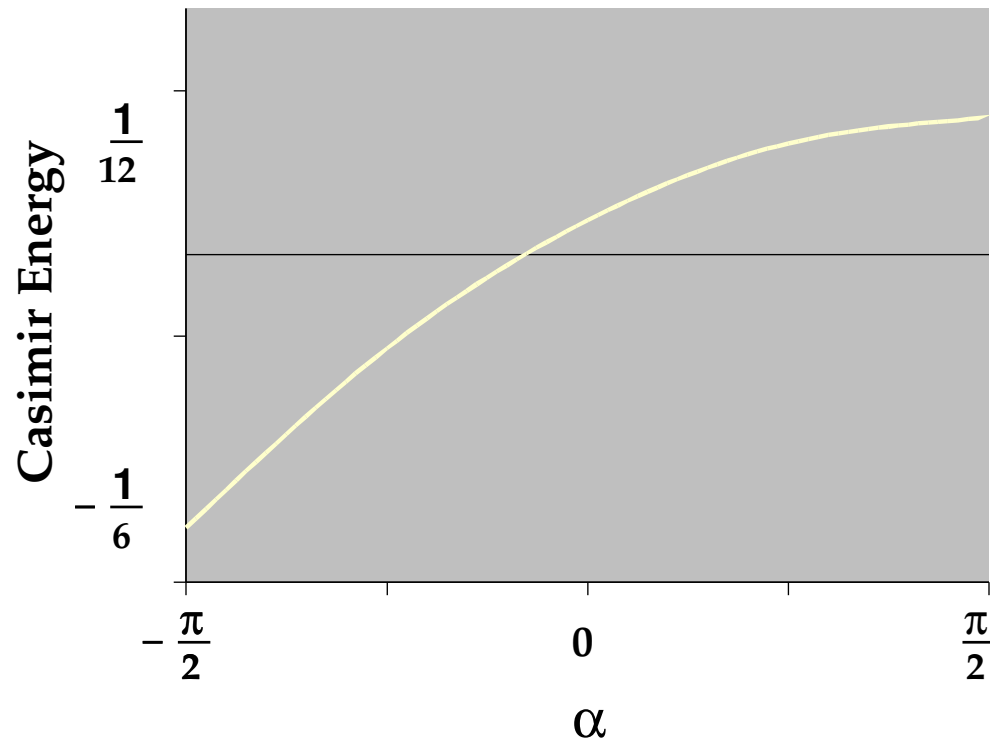
$$E_0 = \frac{\pi}{L} \left(\frac{1}{12} - \left(\frac{\alpha}{2\pi} - \frac{1}{4} \right)^2 \right)$$

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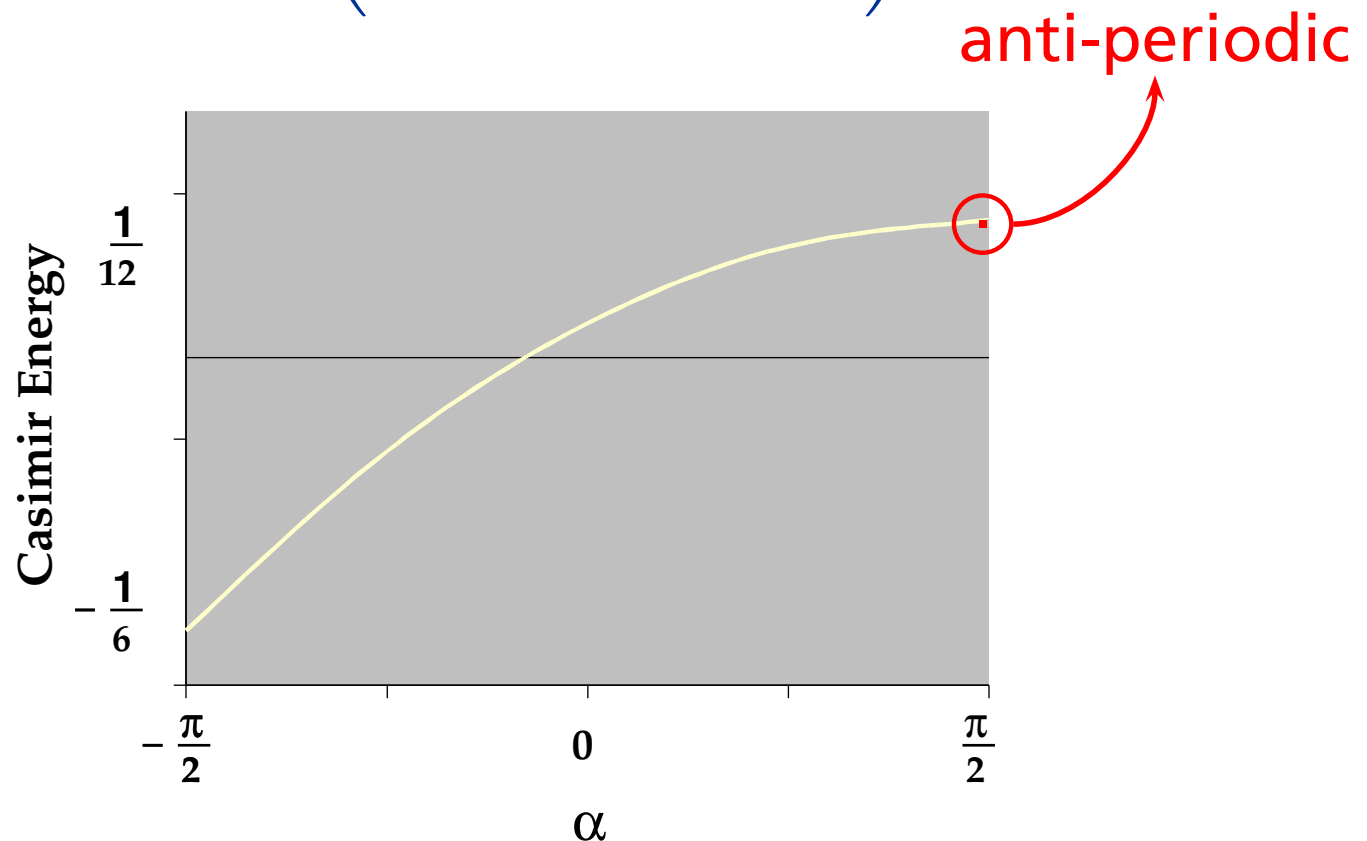


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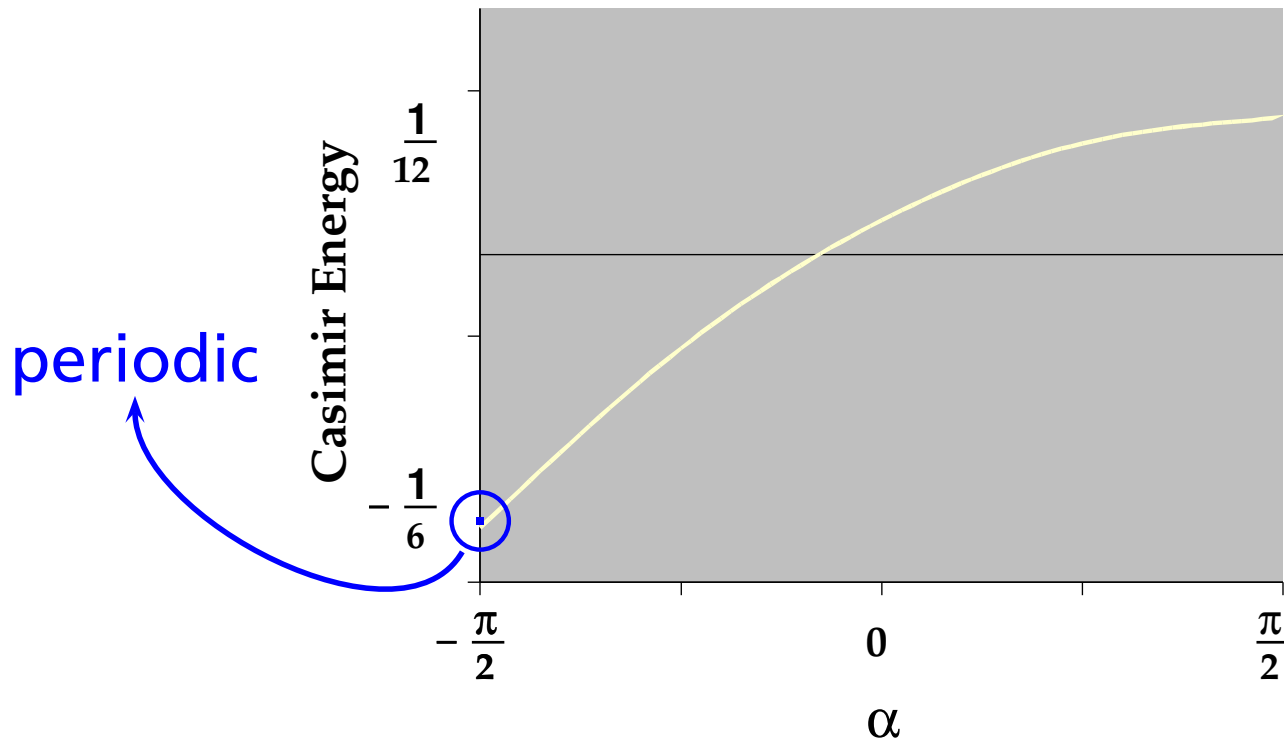


Casimir Energy: Quasi-periodic b.c.

$$U = \cos \alpha \sigma_z - \sin \alpha \sigma_x$$

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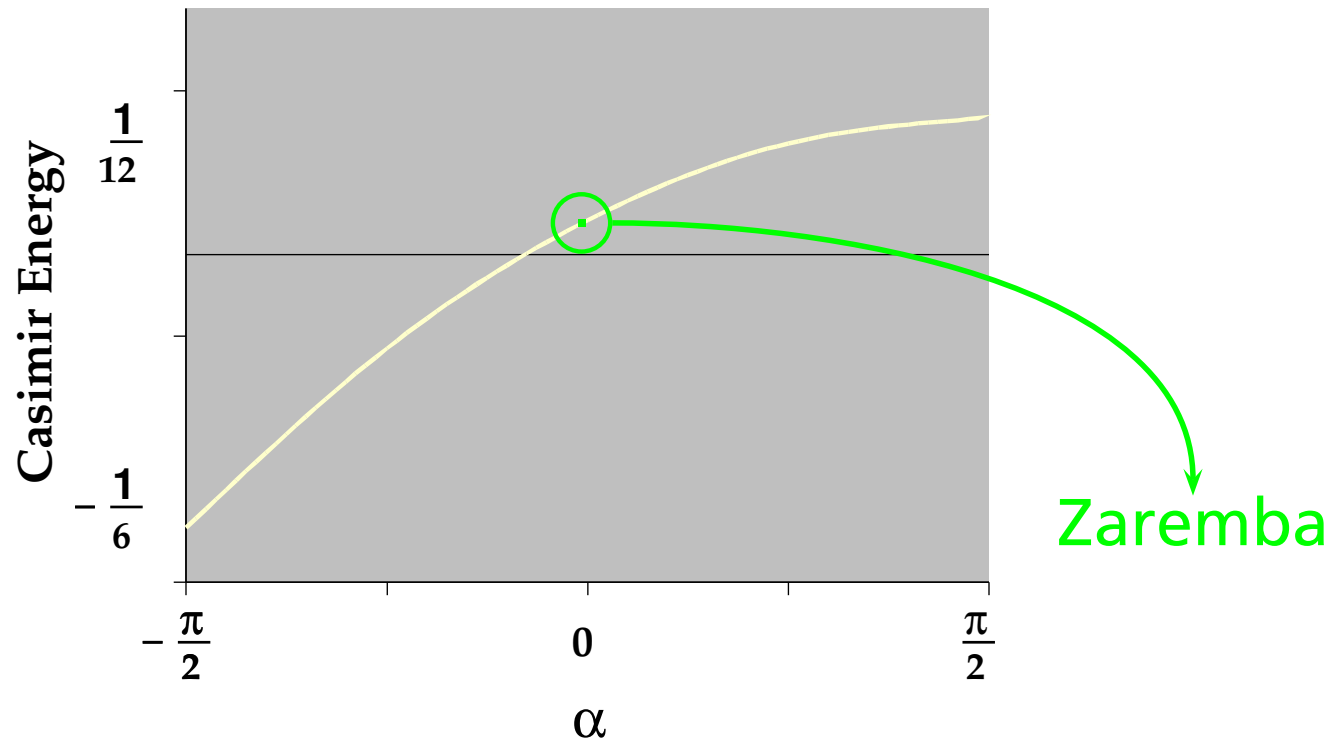


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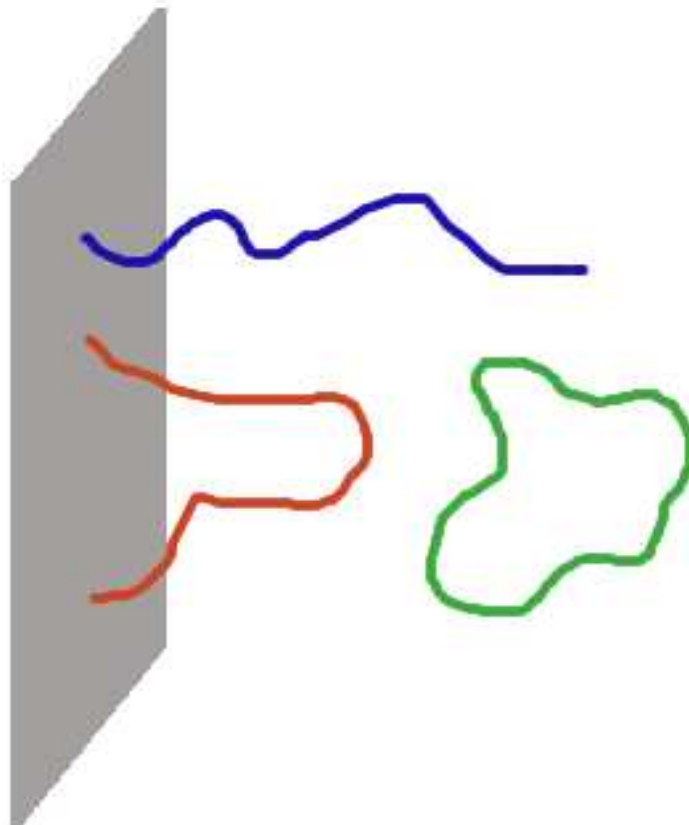
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Strings and D-branes

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- Cyclic fixed points appear only with singular boundary condition (Russian doll)