

Ghosts conditions for GB and MGM cosmologies

Antonio De Felice

University of Sussex

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with Mark Hindmarsh, Mark Trodden ([astro-ph/0604154](#))

with Gianluca Calcagni, Beatriz de Carlos ([hep-th/0604201](#))



Introduction

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 - ★ or non-minimally coupled scalar field?

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- ★ Modify gravity at large scales [Carroll, ADF, Duvvuri, Trodden, Turner, PRD]

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 - ★ Powers of curvature invariants \rightarrow ghosts
 - ★ **No** 4th derivatives for the Gauss-Bonnet term
 - ★ Possible non-minimal couplings bw ϕ and R_{GB}^2

GB – The Model II

- The action under study

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f_1(\phi) R - \frac{1}{2} \omega(\phi) (\partial\phi)^2 - V(\phi) + f_2(\phi) R_{\text{GB}}^2 \right]$$

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- It leads, in general, to a system of 2 non-linear ODEs of 2nd order in a and ϕ in FRW.

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- Simple case of a single modulus
 - ★ $f_1 = 1$, $\omega = 3/2$, $U = 0$, $f_2 \approx \cosh(\phi)$
- In general $F = \beta e^{\gamma\phi}$, U non-perturbative.

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- Particle content: Graviton + ϕ + matter

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- **Not** all backgrounds may be safe

Cosmological perturbations

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- Cosmology needs FRW, inflation
- Expand the action about general flat FRW!

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$$S = \int dt a^3 \left[-A(t) \Psi \ddot{\Psi} + \frac{B(t)}{a^2} \Psi \nabla^2 \Psi \right]$$

- Speed of propagation

$$s = \frac{B(t)}{A(t)}$$

Ghosts and instabilities

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- Vector modes do not propagate [Hwang, Noh PRD 61 (2000)]

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- Instability: No wave-propagation, Euclidean box, exponential behaviour, $AB < 0$
- Superluminal: No unicity of the future cone, ill-defined Cauchy problem, $B/A > 1$

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- Ghosts instabilities, no FRW

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- Unless $f_2(\phi) \rightarrow 0$

Constraints from initial conditions

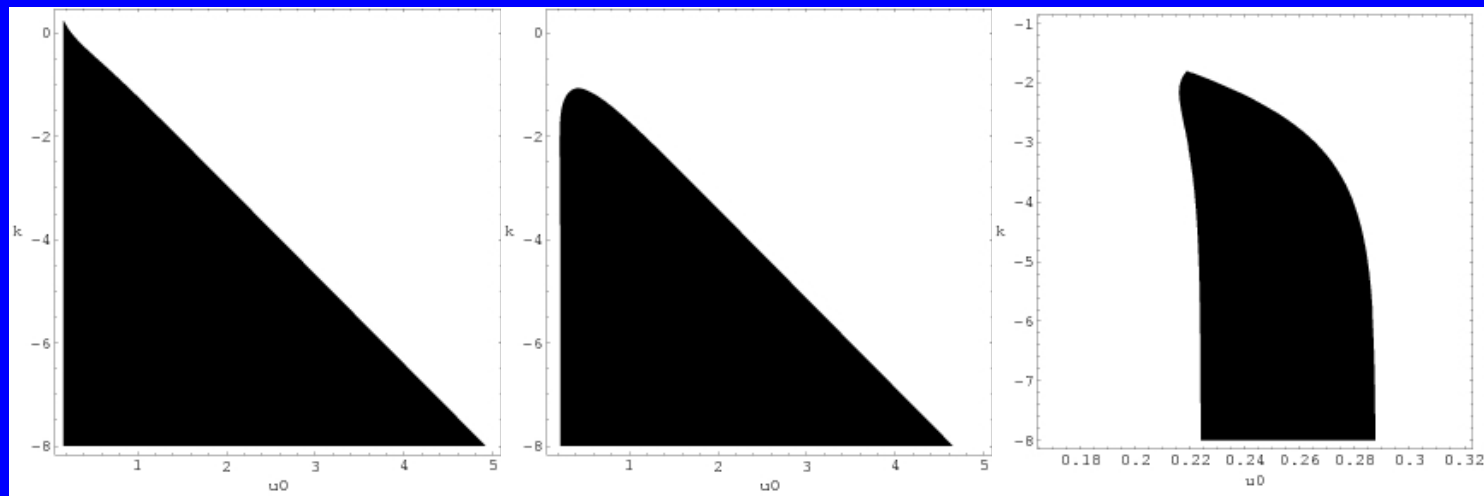
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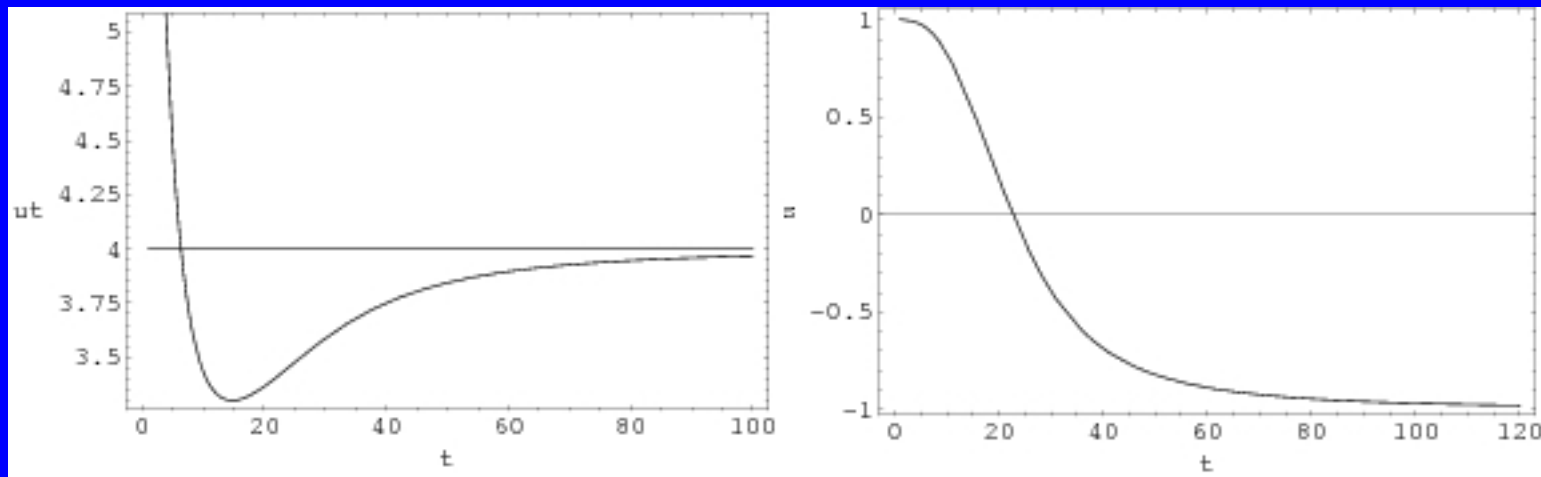


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- Need to change effective action