NUCLEI, CHAOS AND STATISTICAL SPECTROSCOPY

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Introduction

- NUCLEI $\longrightarrow$ COMPLEX MANY BODY SYSTEMS
- Describe them by 'AVERAGED' properties + 'FLUCTUATIONS' around them
- AVERAGE: Level density, Spectra, Orbit Occupancy, Transition Strengths and Strength Sums, ...
- FLUCTUATIONS: Nearest neighbour spacing distribution (NNSD), Δ_3 Statistics,.......
- Describe the Nuclear Hamiltonian by RANDOM MATRICES - satisfying certain symmetries (J,T),....
- (N X N) Hamiltonian: diagonalise $\longrightarrow$ Energy Levels $\longrightarrow$ Level Density $\rho(E)$

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- Describe the Nuclear Hamiltonian by RANDOM MATRICES - satisfying certain symmetries (J,T),....
- (N X N) Hamiltonian: diagonalise → Energy Levels → Level Density $\rho(E)$
- PREDICTIONS of theory based on random matrices (Spectral Distribution Methods) Do they agree with experimental data/shell model ?
- FLUCTUATION properties - do they agree with data?
- Fluctuation Properties of the Random Matrix Ensemble (GOE) Universal
- "Bohigas-Giannoni-Schmit conjecture' (1984) - Link between Random Matrix Theory and Spectral Fluctuation Properties of Chaotic Quantum Systems

Nuclear Shell Model

- Effective Nuclear Hamiltonian $H = H_0 + H^{'}$
- 1-body part :

$$H_0 = \Sigma \epsilon_\lambda a_\lambda^\dagger a_\lambda$$

where ϵ_λ are the single particle energies

- Describes non-interacting fermions in the mean field with an appropriate core
- Example: $^{20}Ne_{10}$

Core $^{16}O_8$: So 4 valence nucleons in sd-shell (T_z =0) i.e. the three orbits $d_{5/2}$, $d_{3/2}$ and $s_{1/2}$

- The m-particle basis states are antisymmetrised product of the s.p. staes $|\lambda >$
- Residual Interaction:

$$H^{'} = \Sigma < kl|H|ij > a_l^\dagger a_k^\dagger a_i a_j$$

giving rise to nondiagonal matrix elements

- PROCEDURE:

i) Construct all the m-particle basis states with (J,T) as good quantum no

ii) Construct all the matrix elements of H in this m-particle basis with fixed (J,T)

- PROCEDURE: (contd)

  iii) Large Matrix For example for $^{28}Si_{14}$ dimensions 3276, 5768 for (J,T)= (2,0) and (2,1) respecively

  iv) DIAGONALISE this matrix H for each (J,T)

- CONFIGURATION MIXING

$$|E >= \Sigma c_i|\Phi_i >$$

- Each configuration given by $(m_1, m_2, ..., m_l)$ for l orbits
- For large excitation energies one needs to widen the space by involving higher shell orbits

Random Matrix Ensembles

DENSITY OF STATES WITH 1-BODY HAMILTONIAN (J.B. French -1971 to 1975)

Eigenvalues of H in large shell model spaces:

$$H = H(1) = \Sigma \epsilon_i n_i$$

$$E = \epsilon_1 + \epsilon_2 + \epsilon_3 = ...$$

- Ignore Pauli blocking: USE CENTRAL LIMIT THEOREM (CLT)

$$\rho(E) \rightarrow Gaussian$$

- Only 2 moments are needed: $\varepsilon =< \mathbf{H} >= \mathbf{Trace}(\mathbf{H})/\mathbf{Dimension}$ and $\sigma^2 =< H^2 > -\varepsilon^2$
- If one adds a 2-body Hamiltonian to this one needs to define Ensembles of Random H's and average over the ensembles for the Averaged Density of States
- French and Smith:

  The envelope of the 1-body H's density of states for a number of shells gives roughly the Bethe's level density formula given by $C \times exp(2[aE]^{1/2}/(E^2)$

GOE

Gaussian Orthogonal Ensemble in the space of Real Stmmetric Matrices defined by:

- 1. The ensemble invariant under every tranformation

$$H \rightarrow W^T H W$$

  where W is any real orthogonal matrix

- 2. The various elements $H_{kj} (k \leq j)$ are statistically independent random distributions

- Joint Prob. Dist. of the eigenvalues: (M.L.Mehta- Random Matrices):

$$P_N(E_1, E_2, ...E_N) = C \times exp(-(\Sigma_i E_i^2)) \times \Pi_{ij} |E_i - E_j|$$

- Real Symmetric $H = \Sigma_{\alpha,\beta} W_{\alpha,\beta} \Psi^\dagger_\alpha(m) \Psi_\beta(m)$ where $\Psi_\beta(m) = a_m a_n a_o ... a_l$ (m times)

- $W_{\alpha\beta}$ independent with $\bar{W_{\alpha\beta}} = 0$ and $\bar{W^2_{\alpha\beta}} = (1 + \delta_{\alpha\beta}) v^2$

- RESULT (Wigner):

$$\bar{\rho}(E) = (4 - E^2)^{1/2} / (2 \times \pi)$$

Embedded GOE or EGOE

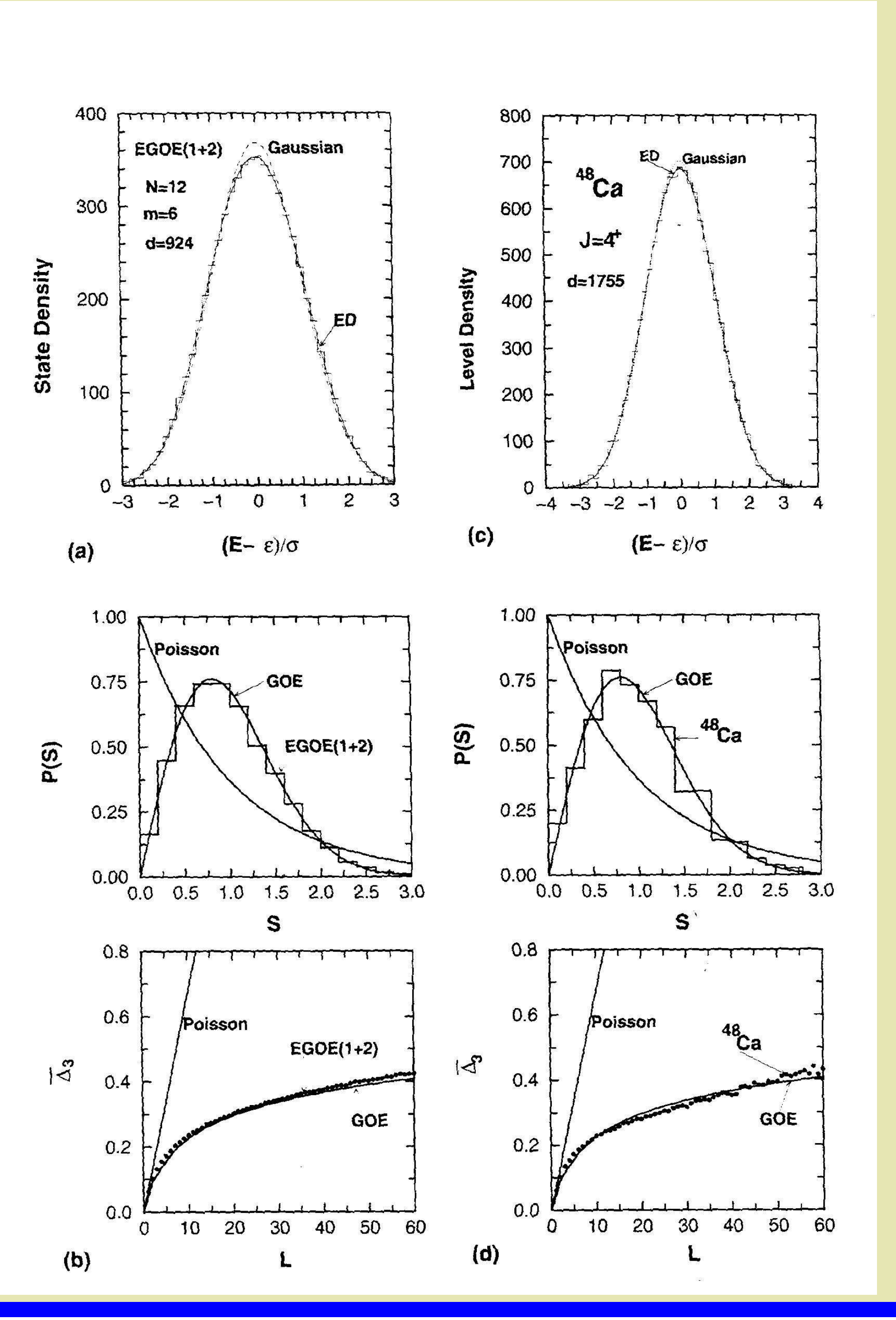
- m-particle space with k-body Hamiltonian

$$H = \Sigma W_{\alpha\beta} \Psi^{\dagger}_{\alpha}(k) \Psi_{\beta}(k)$$

- with the constraints $\bar{W_{\alpha\beta}} = 0$ and $\bar{W^2_{\alpha\beta}} = (1 + \delta_{\alpha\beta}) v^2$
- k=m gives back GOE
- k=2 gives TWO BODY RANDOM ENSEMBLE (TBRE)
- Dilute Limit : m large and N large but $m/N \rightarrow 0$
- Matrix Dimension,d = N(N-1)/2 and no of independent m.e. =d(d+1)/2
- RESULT: m-particle space eigenvalue density $\rightarrow$ GAUSSIAN

$$M_{2\nu} = \Sigma < H H \bar{H} H H H .... > = (2\nu)! < \bar{H}^2 > / (2^{\nu} \nu!) = (2\nu - 1)!! M_2$$

- This is the Moment relation for A Gaussian


EGOE(1+2)
Gaussian
N=12
m=6
d=924
ED
State Density
(E− ε)/σ
(a)
48Ca
J=4+
d=1755
Level Density
(c)
Poisson
GOE
EGOE(1+2)
P(S)
S
48Ca
Δ̄3
L
(b)
(d)

Spectral Distribution Methods: Propagation of Traces

- Scalar (m) Space:

$$TrG(k) = \quad << G(k) >>= \quad {}^{N-k}C_{m-k} \quad << G(k) >>$$

or

$$< G(k) >^{m}= \quad {}^{m}C_{k} \quad < G(k) >^{k}$$

U(N): unitary transformation in the space of N single particle states

$$< H >^{m}= \quad m\,\bar{\varepsilon} +^{m} C_2 \;\; \bar{W}$$

where $\bar{\varepsilon}$ is the average single particle energy and $\bar{W}$ is the average 2 body energy

- Scalar-Isospin (m,T) Space:

$$\begin{aligned} < H >^{m,T}= \quad & m\,\bar{\varepsilon} + (1/8) \times [m(m+2) - 4T(T+1)] \times W^{0} \\ & +(1/8) \times [3m(m-2) + 4T(T+1)] \times W^{1} \end{aligned}$$

- Configuration-Isospin Space:

$$< H >^{\bar{m},T}= \quad fn.\;\; of \;\; [m_i, T_i^2, T_i.T_j]$$

- Similarly write averages for $< \; F \;\; G \; >^{m}$, $< \; F \;\; G \; >^{m,T}$ and their correlation coefficient.

Chaos in Quantum Systems: Nuclei

Signatures of Chaos

- In quantum systems with discrete energy spectrum E_i define $$S_i = (E_{i+1} - E_i)/\bar{D}$$
- A). Nearest Neighbour Spacing Distribution:
- CHAOS $\rightarrow P_{Wigner} = \pi \times (S/2) \times exp^{-(\pi/4)S^2} \rightarrow (GOE)$
- REGULAR $\rightarrow P_{Poisson} = C \times exp(-S)$
- Division by $\bar{D}$ is known as unfolding (rescaling) the spectrum Very Important
- For numerical studies one uses: $P_{brody}(S) = \alpha(w+1)S^w exp(-(\alpha S^{w+1}))$ (w=1 gives Wigner and w=0 gives Poisson)
- B). Δ_3 Statistics

$$\Delta_3(L) = (1/L) min_{(A,B)} \int_{L/2}^{L/2} [N(E) - AE - B]^2 dE$$

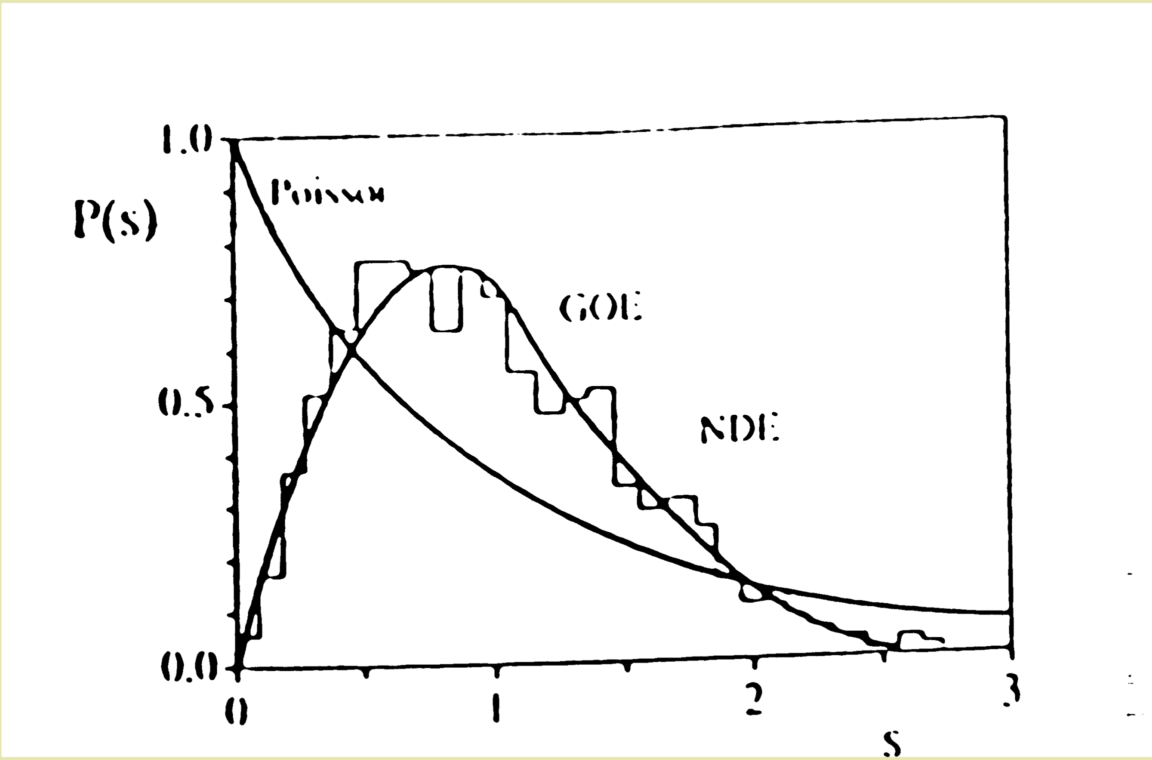
- in a fixed interval (-L/2, L/2) least square deviation of the staircase fn N(E) (related to $\rho(E)$) from the best linear fit
- CHAOTIC $\rightarrow \Delta_3(L) = logL/\pi^2$
- REGULAR $\rightarrow \Delta_3(L) = L/15$

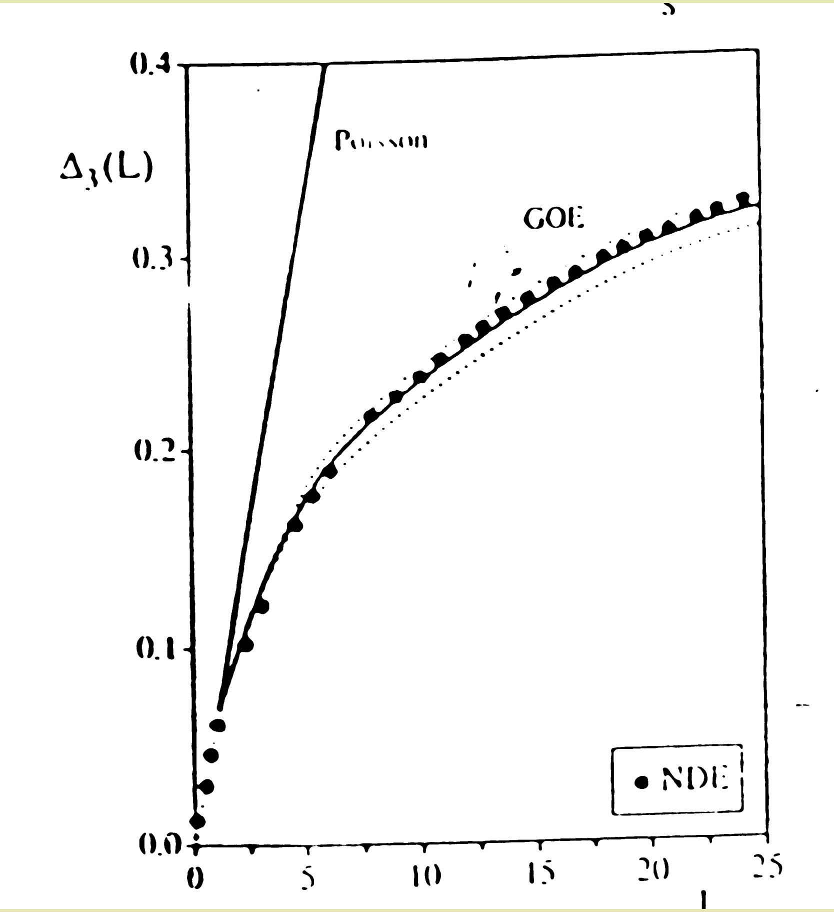
GOE with d=2

- $H_{11} = G(0, 2v^2)$, $H_{22} = G(0, 2v^2)$, $H_{12} = G(0, v^2)$
- $E_{1,2} = (H_{11} + H_{22})/2 +_{-} ((H_{11} - H_{22})^2 + 4H_{12}^2)^{1/2})/2$
- $s^2 = (H_{11} - H_{22})^2 + 4H_{12}^2$
- s=0 needs $H_{11} = H_{22}$ and $H_{12} = 0$
- $$\begin{aligned} P(H)dH &= exp(-(H_{11}^2 + H_{22}^2 + 2H_{12}^2)/4v^2))dH_{11}dH_{22}dH_{12} \\ &= exp(-(Tr(H^2)/4v^2))dH_{11}dH_{22}dH_{12} \end{aligned}$$
- Transform $x_1 = (H_{11} - H_{22})/2$, $x_2 = H_{12}$ and $x_3 = (H_{11} + H_{22})/2$ and then change $(x_1, x_2) \longrightarrow (s, \theta)$
- $P(s)ds = Cexp(-s^2/8v^2)sds$

In Nuclei:

- Nuclei excited by a few MeV behave as Chaotic Quantum Systems
- First observed by Haq, Pandey and Bohigas (1982) in Nuclear Data Ensemble -using 1726 spacing of levels with same spin and parity, much above ground state, obtained from neutron time-of-flight spectroscopy and from high resolution proton scattering



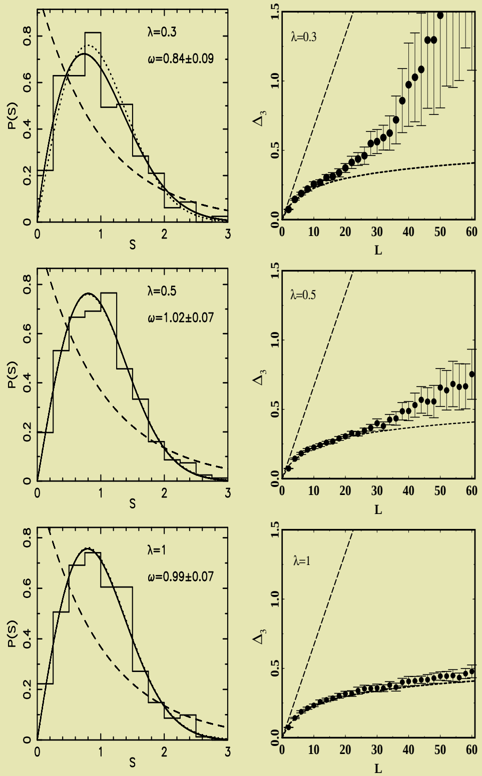


- Question: A) At what energy transition to chaos takes place?
- Question: B) What parameters of H decide that ?
- Many analytical studies done with model Hamiltonians

- Also numerically construct $H_\lambda = H_0 + \lambda V = H_0 + \lambda(H_{SM} - H_0)$
  At what value of λ does transition to chaos happen ? Results later

- With a H prserving a group symmetry- SU(4)ST group
  $$H_\lambda(SU(4)) = H_{SU(4)-ST scalar} + \lambda(H_{SM} - H_{SU(4)-ST scalar})$$

Nearest Neighbour Spacing Distribution and Δ_3 as a function of λ (Here $H_0 = \Sigma n_i \epsilon_i$)



Other Measures of Chaos

$$|E >= \Sigma_k C_k^E \quad |k >$$

where C_k^E are the amplitudes in the expansion

As a measure for the degree of the complexity of individual wavefns define the Information Entropy:

$$\mathrm{S}^{info}{}_E \;=\; -\Sigma_k \;\left|C_k^E\right|^2 \;\ln\; \left|C_k^E\right|^2$$

and Localisation Length:

$$l_H(E) \;=\; exp\left[\mathrm{S}^{info}{}_E\right]/0.48\; d$$

$0.48 \times d$ is the GOE value for S^{info}. Thus $l_H(E)$=1 for GOE.

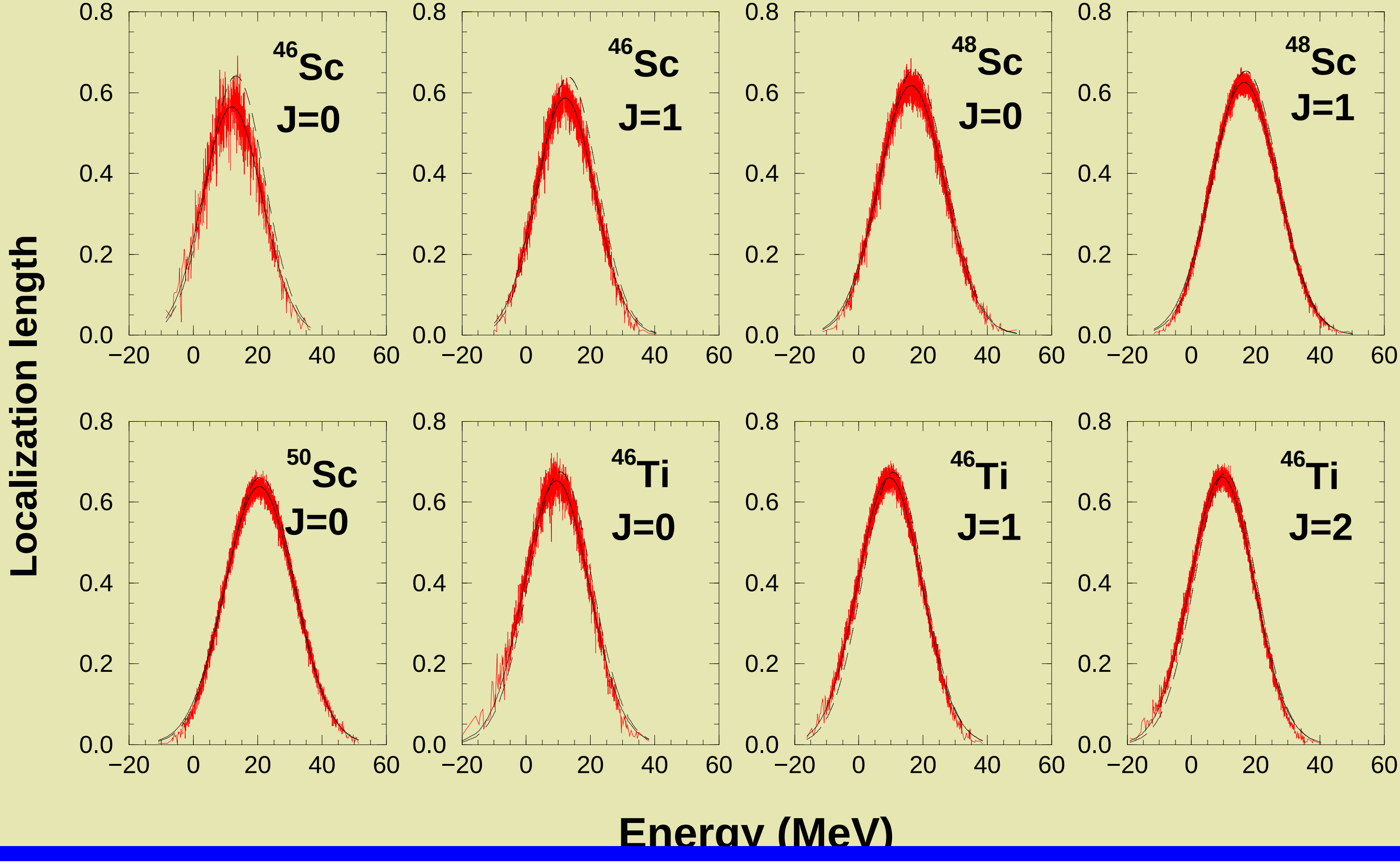
One also defines the Number of Principal Components, NPC as

$$(\mathrm{NPC})_E \;=\; \left[\Sigma_k \left|C_k^E\right|^4\right]^{-1} \tag{1}$$

For GOE, $(NPC)_E$ is again independent of energy and equal to $d/3$.

The EGOE results are derived by Kota and Sahu (2001).

Exact shell model results compared with EGOE predictions of Kota and Sahu without and with polynomial correction term



SPECTRAL DISTRIBUTION THEORY RESULTS

Transition Strength Distributions and Strength Sums

- Transition Operator 'O' connecting initial state $|m, E>$ to final state $|m^{'}, E^{'}>$

$$R(E, E^{'}) = | < m^{'}, E^{'}|O|m, E > |^2$$

- Bivariate distribution

$$I^{(m,m^{'})}(E, E^{'}) = I^{m^{'}}(E^{'}) \times | < m^{'}, E^{'}|O|m, E > |^2 \times I^m(E)$$
$$=<< O^{\dagger}\delta(H - E^{'})O\delta(H - E) >>^m$$

where $<< .... >>$ denotes trace

- Bivariate strength density $\rho_s =< O^{\dagger}\delta(H - E^{'})O\delta(H - E) >^m / < O^{\dagger}O >^m$
- Moments $\mu_{pq} =< O^{\dagger}((H - \varepsilon_2)/\sigma_2)^p O((H - \varepsilon_1)/\sigma_1)^q >^m / < O^{\dagger}O >$
- EGOE RESULT: (French, Kota, Pandey, Tomsovic , Ann Phs. 181, 235)

  With k-body H and $k^{'}$-body O and for Dilute case

  The cumulants $k_{rs} \rightarrow o$ for (r,s) $\geq$ 3
- $\rho(E, E^{'}) \rightarrow \rho_{gaussian}(E, E^{'})$

Sum Rule Strengths:

- For tranition operator 'O' connecting intial state $|m, E>$ to final states $|m^{'}, E^{'}>$ Sum Rule Strength

$$K(E) = \Sigma_{E'} R(E, E^{'}) = \Sigma_{E'} < E|O^{\dagger}|E^{'} >< E^{'}|O|E >=< E|O^{\dagger}O|E >=< E|K|E >$$

- $K = O^{\dagger}O$ is the Sum Rule Operator

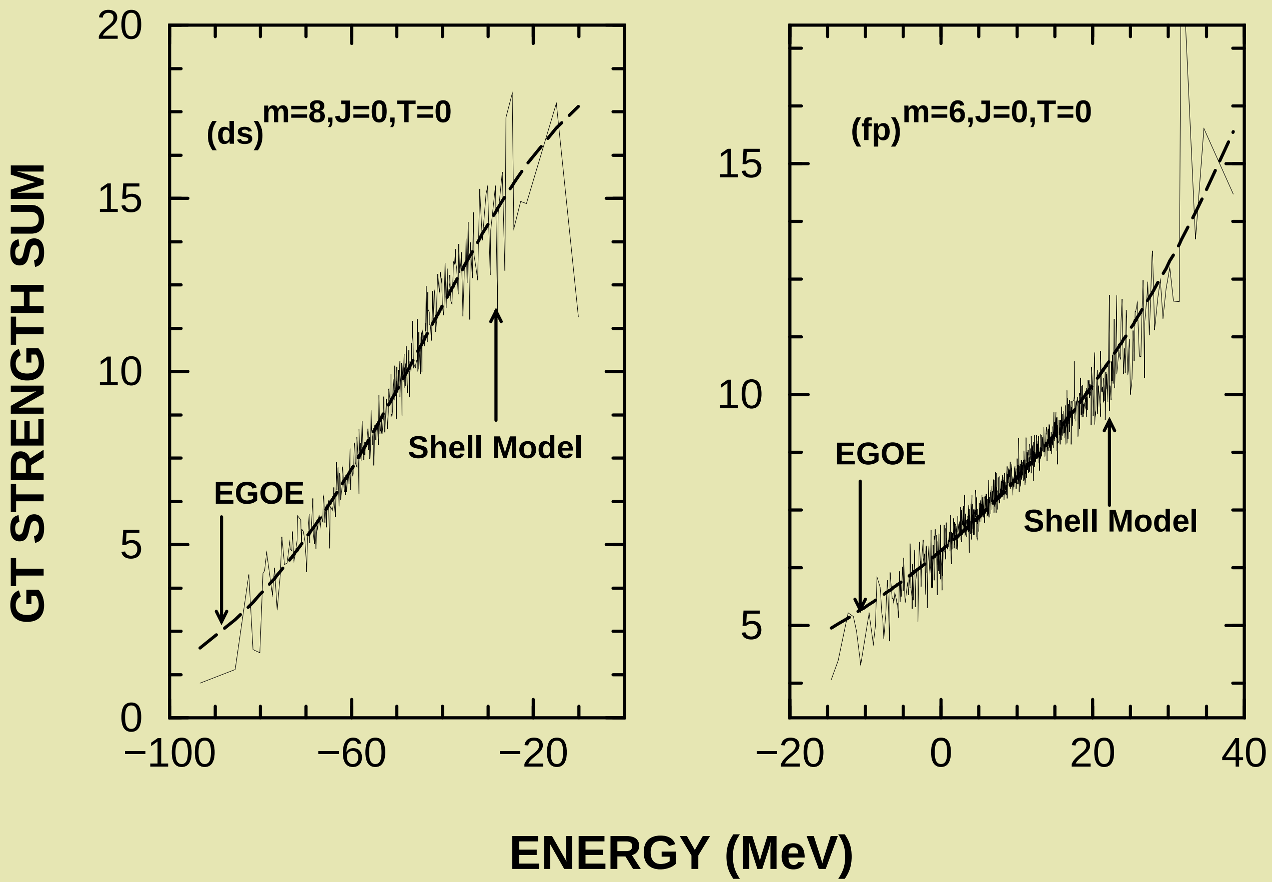
- Examples: a) $O = a_r/a_r^{\dagger}$ K= Occupancy, $a_r^{\dagger}a_r/a_ra_r^{\dagger}$
  b) $O_{GT} = \Sigma_i \vec{\sigma}(i) t_{-}(i)$ β^{-} GT decay $K = O_{GT}^{\dagger}O_{GT}$

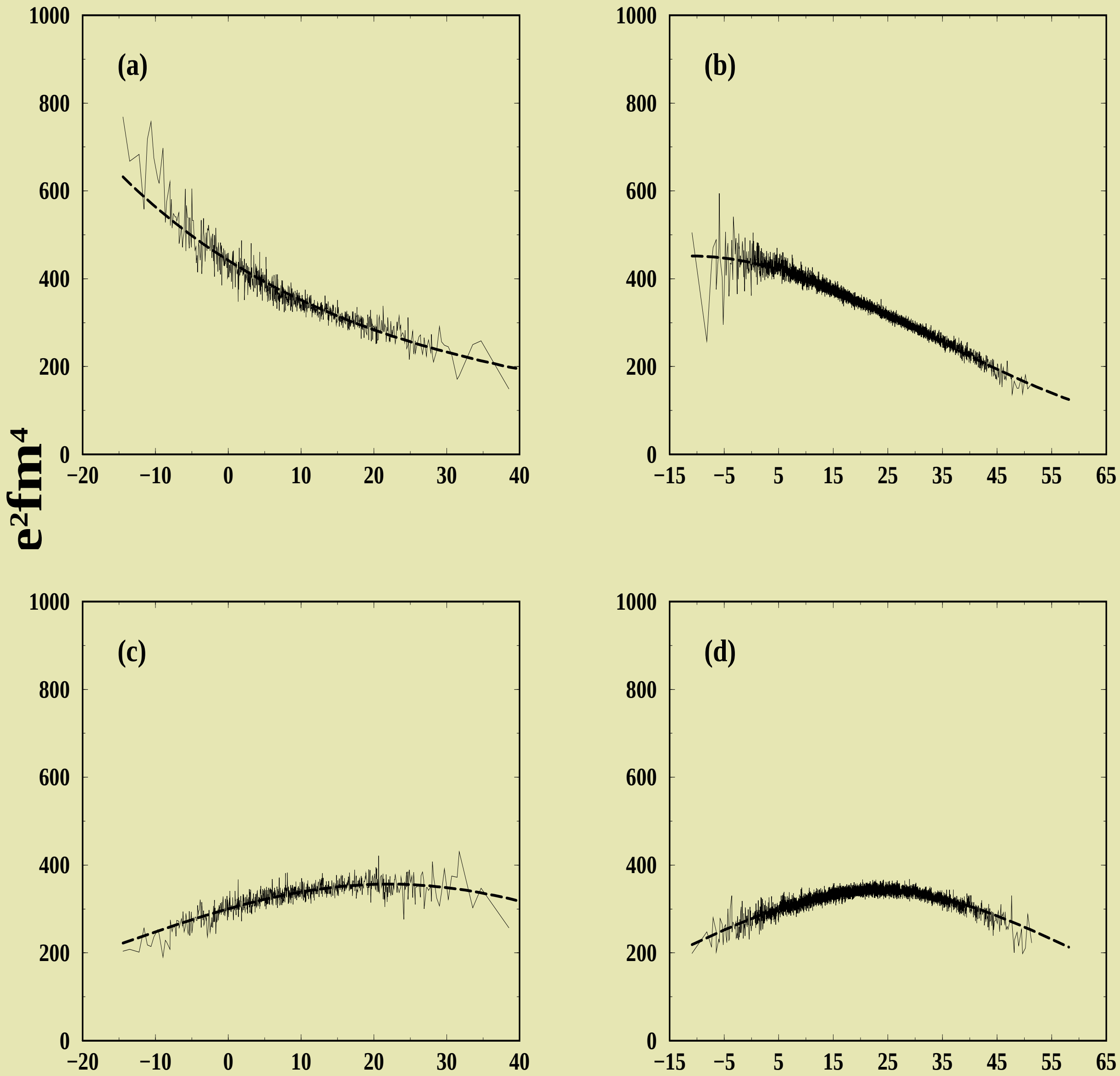
- Using Gaussian forms for Strength density and Energy eigenvalue density
  $M_0(E) =< O^{\dagger}O >^m (\sigma_e/\sigma_s) \times (exp(-(E - \varepsilon_s)^2/2\sigma_s^2)/exp(_(E - \varepsilon_e)^2/2\sigma_e^2))$

- Expressed as the ratio of two Gaussians - so need to calculate the quantities ε_s, σ_s, ε_e and σ_e

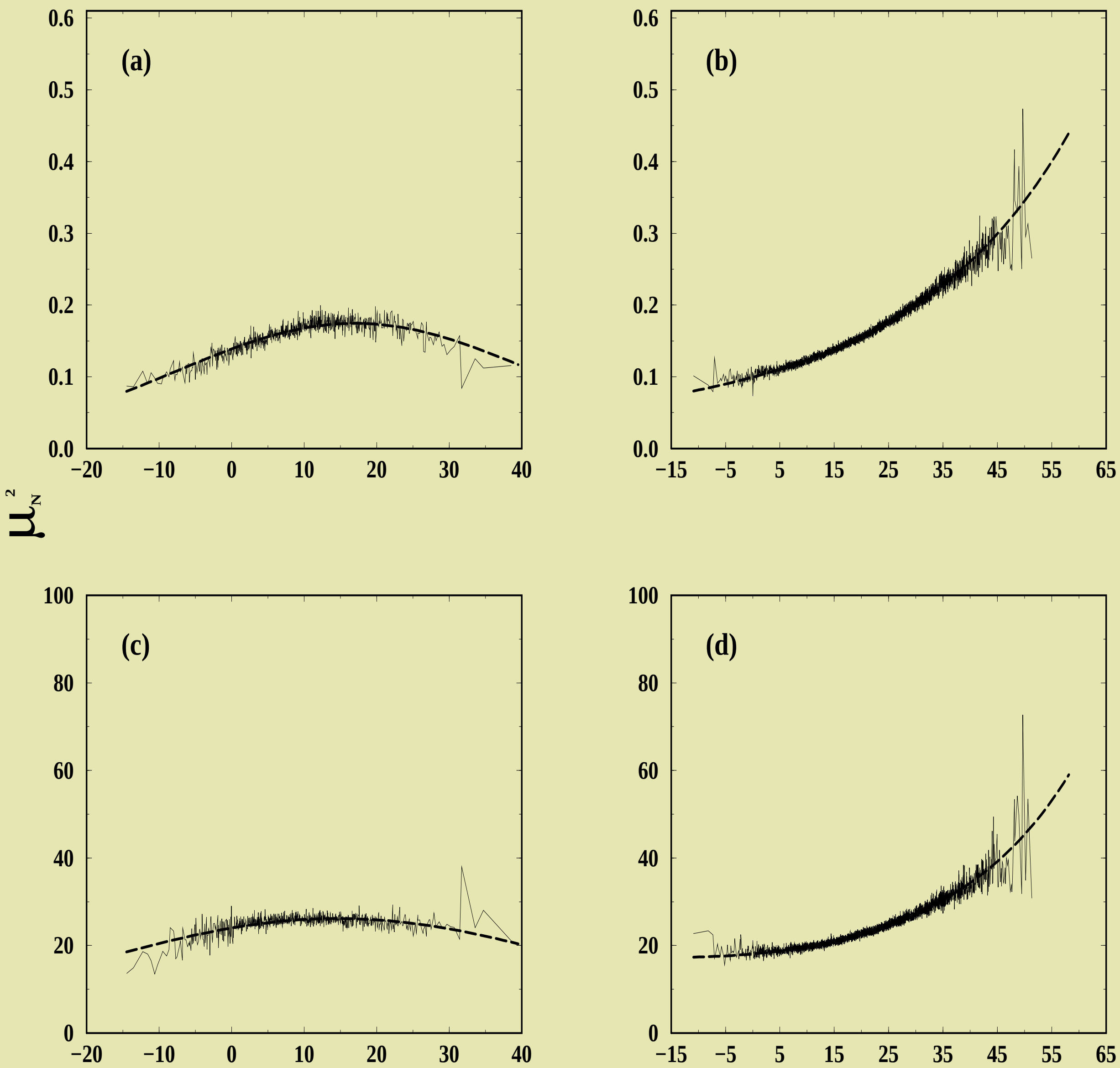
- Computer codes to calculate these based on spectral distribution methods are available

- Detailed check of the applicability has been carried out using the shell model results as input

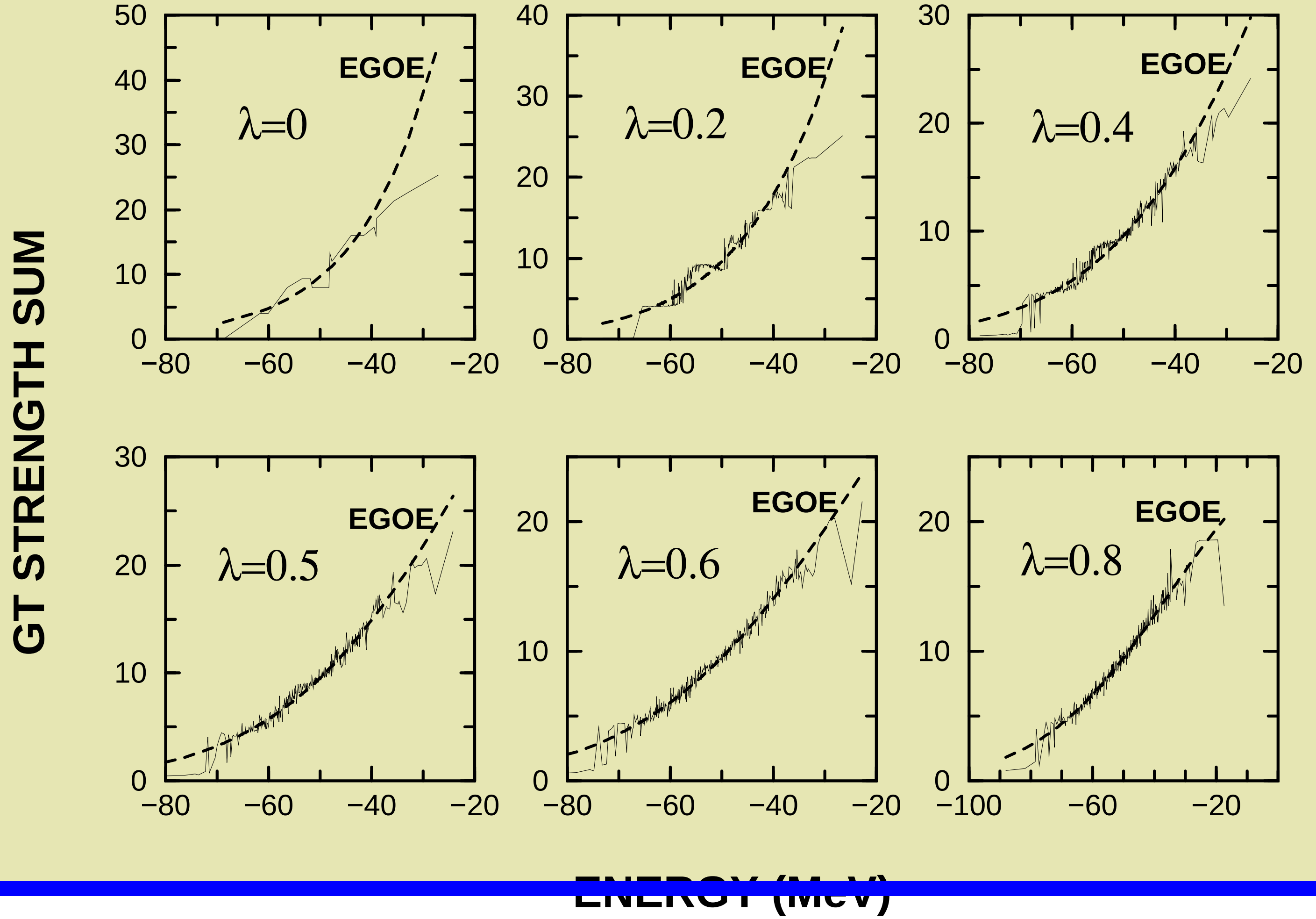

GT STRENGTH SUM
(ds)m=8,J=0,T=0
EGOE
Shell Model
(fp)m=6,J=0,T=0
EGOE
Shell Model
20
15
10
5
0
−100
−60
−20
15
10
5
−20
0
20
40
ENERGY (MeV)



E2 strength sum $\Delta T = 0$ (a) ^{46}V (b) ^{50}Sc $\Delta T = 1$ (c) ^{46}V and (d) ^{50}Sc



M1 strength sum $\Delta T = 0$ (a) ^{46}V (b) ^{50}Sc $\Delta T = 1$ (c) ^{46}V and (d) ^{50}Sc

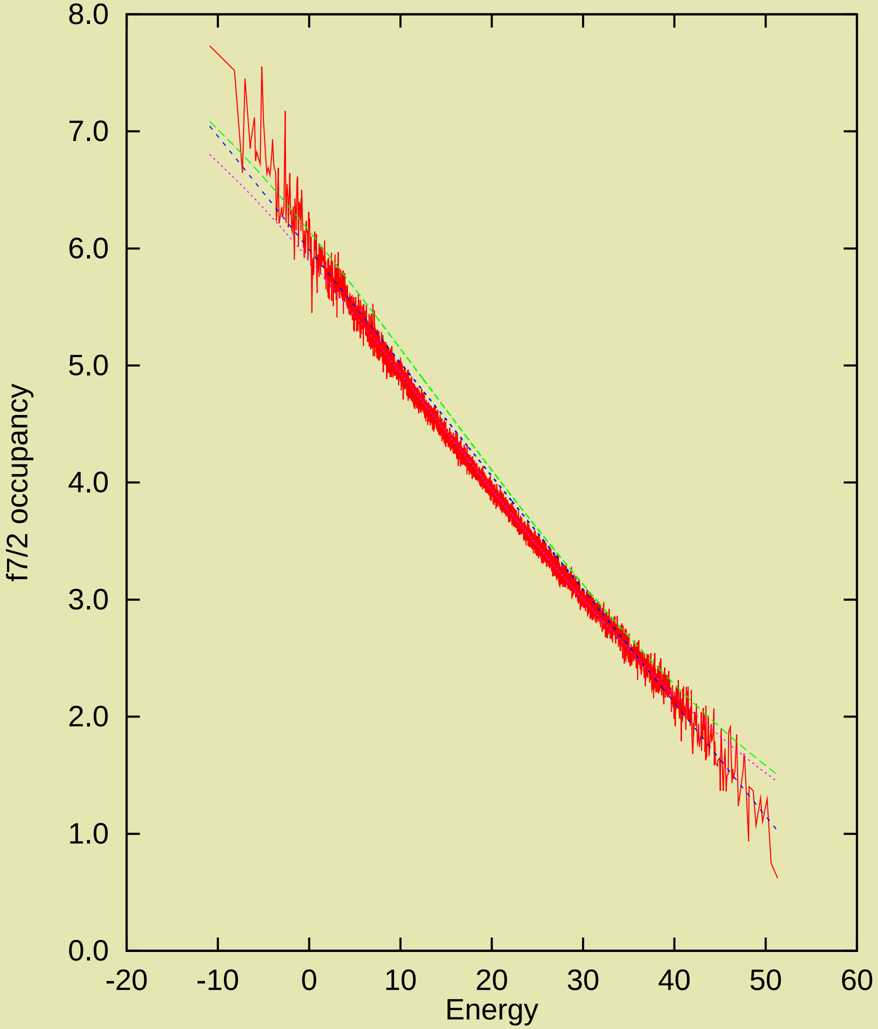


GT STRENGTH SUM
ENERGY (MeV)
EGOE
λ=0
λ=0.2
λ=0.4
λ=0.5
λ=0.6
λ=0.8
0
10
20
30
40
50
−80
−60
−40
−20
−100

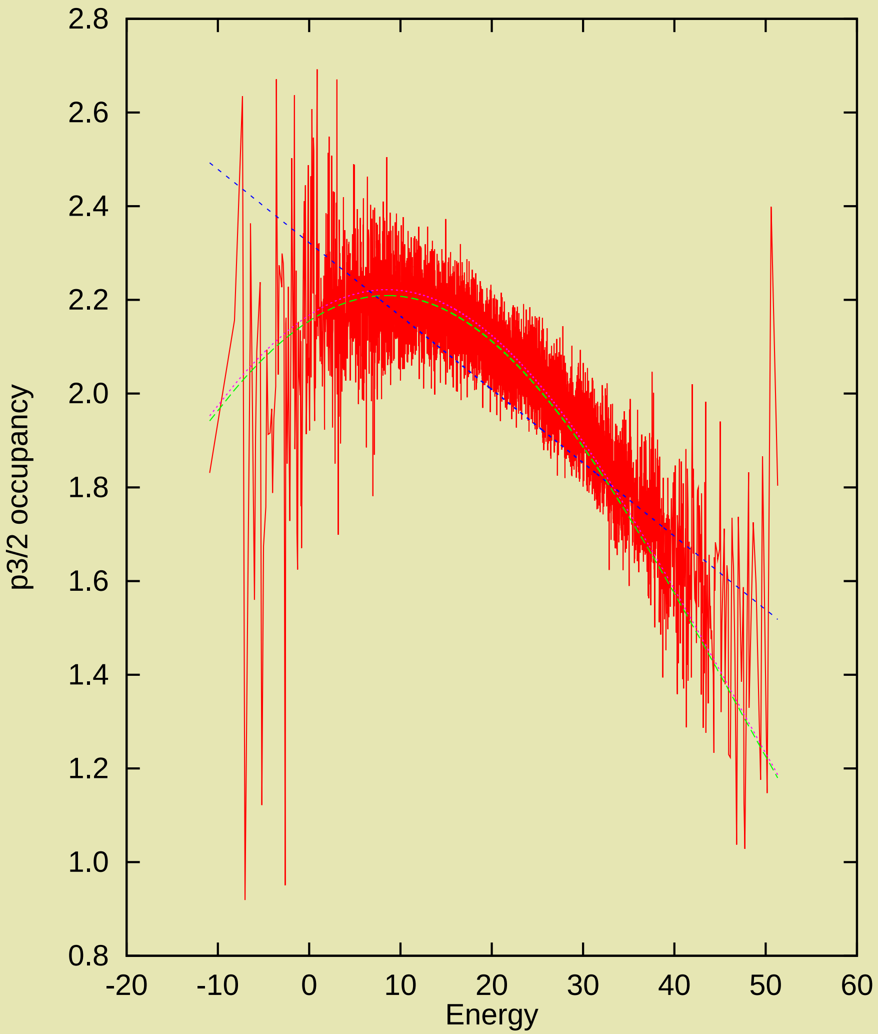
One sees agreement with EGOE around $\lambda = 0.5$. Long Range Order

One can thus use the agreement of Strength Sums with EGOE predictions also as a signature for the setting in of chaos

Occupancy of orbit $f_{7/2}$ in ^{50}Sc by Spectral Distribution Methods



Occupancy of orbit $p_{3/2}$ in ^{50}Sc by Spectral Distribution Methods



SUMMARY

- ♣ RANDOM MATRIX ENSEMBLES VERY USEFUL IN NUCLEAR PHYSICS
- ♣ FLUCTUATION MEASURES OF GOE UNIVERSAL - USED TO STUDY ONSET OF CHAOS IN NUCLEI
- ♣ EGOE MORE REALISTIC FOR NUCLEAR SYSTEMS - SOME RESULTS FOR THE MEASURES OF CHAOS BASED ON MANYPARTICLE WAVEFUNCTIONS DEVELOPED
- ♣ SPECTRAL DISTRIBUTION THEORY PREDICTS ASYMPTOTIC FORMS FOR AVERAGED QUANTITIES -GOOD AGREEMENT WITH SHELL MODEL
- ♣ NUCLEI IN THE CHAOTIC DOMAIN VERY WELL DESCRIBED BY EGOE AND SPECTRAL DISTRIBUTION THEORY
- ♣ MORE APPLICATIONS OF THIS STATISTICAL SPECTROSCOPY ENVISAGED