

# Volodymyr Magas

## Introduction to relativistic heavy ion collisions

Part 4

Dec 10, 2008

# Outline

## *Puzzles in ultra-relativistic heavy ion collisions*

- Quark number flow scaling at RHIC (Lecture 3)
- HBT puzzle
- Experimental FO condition

## *New ideas in ultra-relativistic heavy ion collisions*

- Color Glass Condensate
- Thermal Hadronization and Hawking-Unruh Radiation
- Quark Gluon Plasma as a matter with the lowest possible viscosity in the Universe
- Particle Number Fluctuations - nonequivalence of different ensembles

# HBT Puzzle

# Two particle interferometry

Two particle (boson) correlations are sensitive to the space-time geometry of the source

HBT - in astrophysics

$$C_2(p_1, p_2) = C(q, K) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}$$

where  $q^\mu = p_1^\mu - p_2^\mu$  and  $K^\mu = (p_1^\mu + p_2^\mu)/2$

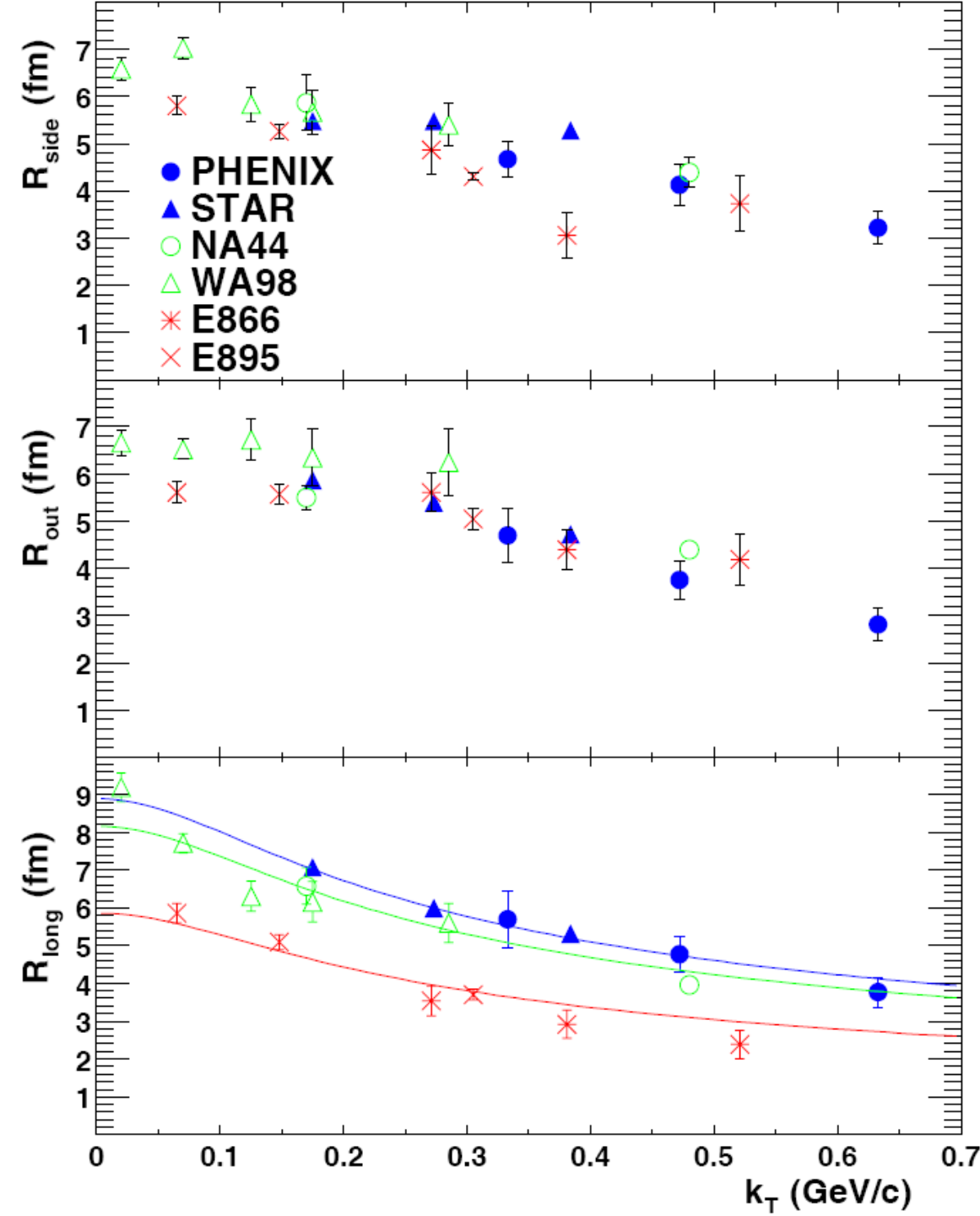
Pratt-Bertsch parameterization

$$C(q_{Out}, q_{Side}, q_{Long}) = 1 + \lambda e^{-(q_{Out}^2 R_{Out}^2 + q_{Side}^2 R_{Side}^2 + q_{Long}^2 R_{Long}^2)}$$

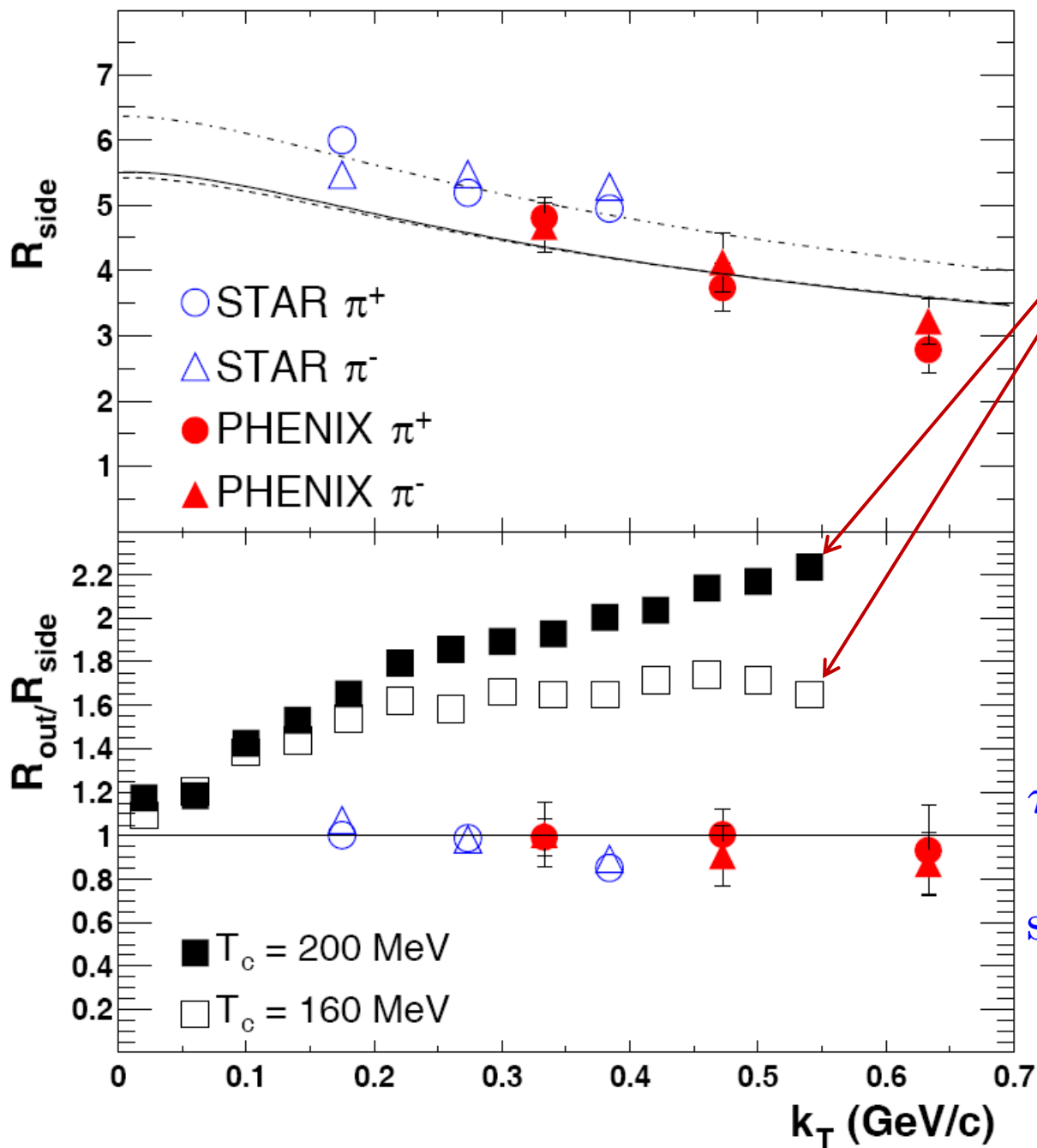
- $R_{Long}$  - connected to system size in beam direction
- $R_{side}$  - connected to transverse system size
- $R_{out} = \sqrt{R_{side}^2 + \tau^2 \beta_T^2}$ , where  $\tau$  is the lifetime of the source, i.e. pion emission time

# HBT Puzzle

1) HBT radii are the same for SPS and RHIC



# HBT Puzzle



Theoretical Calculations with gradual FO

2)  $R_{out}/R_{side} \sim 1 \Rightarrow$

$\tau \rightarrow 0$  - flash-like emission, sharp freeze out

# HBT Puzzle

## Two possible solutions

1) HBT measures not the total space-time volume of the source, but some correlation volume, which depends of the properties of media.

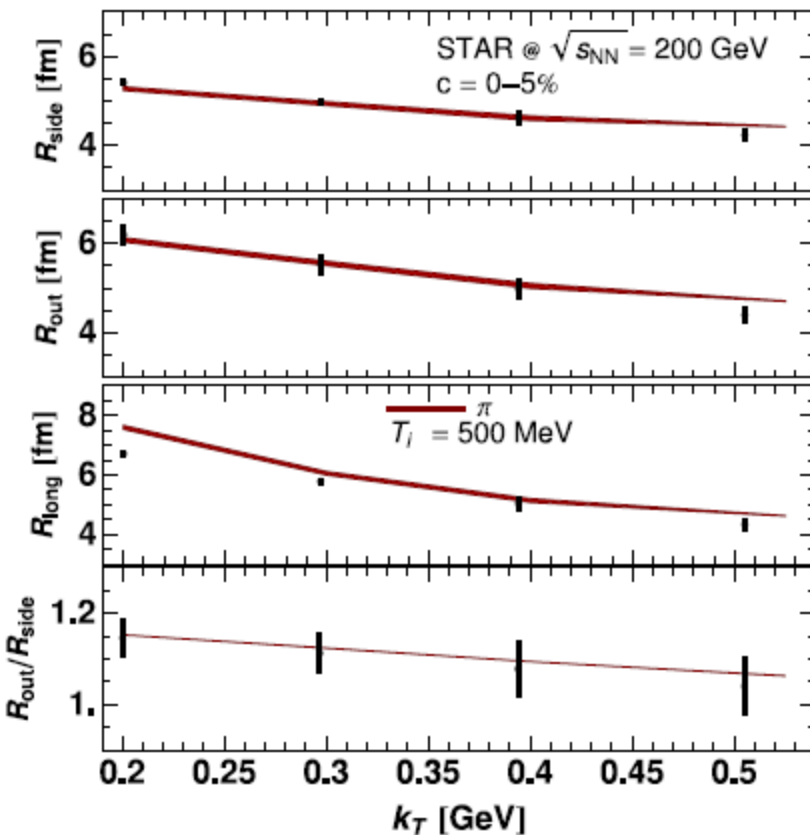
At SPS energies system size is already bigger than the correlation volume → HBT will show the same radii for more energetic AA collisions

2) HBT puzzle might be resolved based on same special initial state and FO hypersurface.

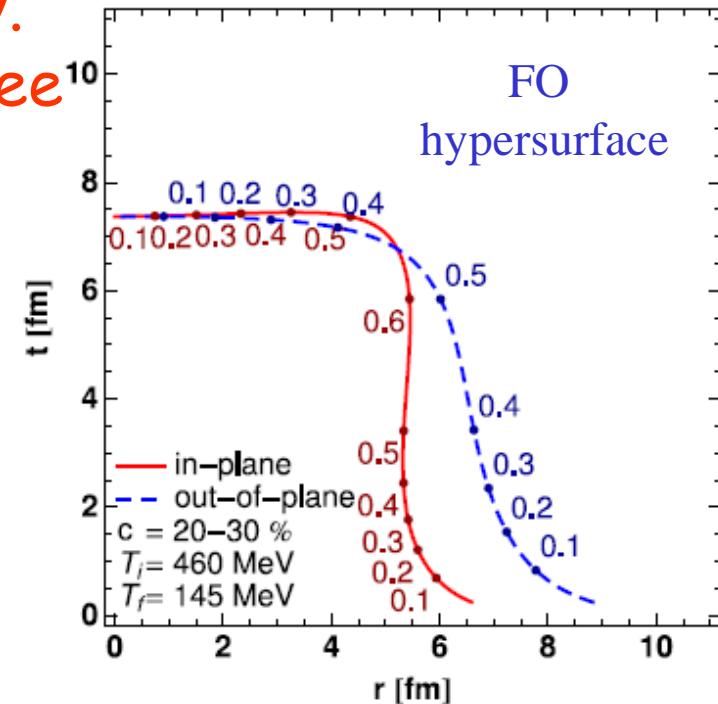
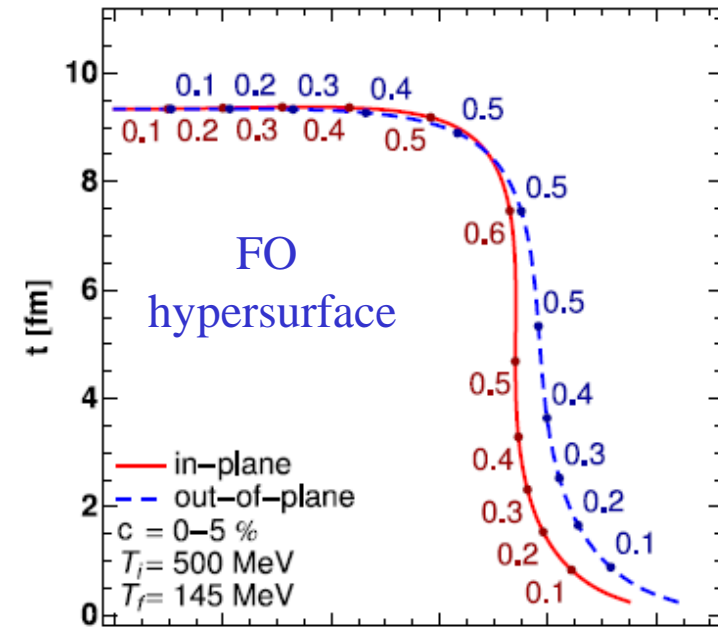
Gaussian initial density profile

$$n(x, y) = \exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}\right)$$

Results



2+1D  
 boost inv.  
 baryon-free  
 hydro



Experimental FO condition

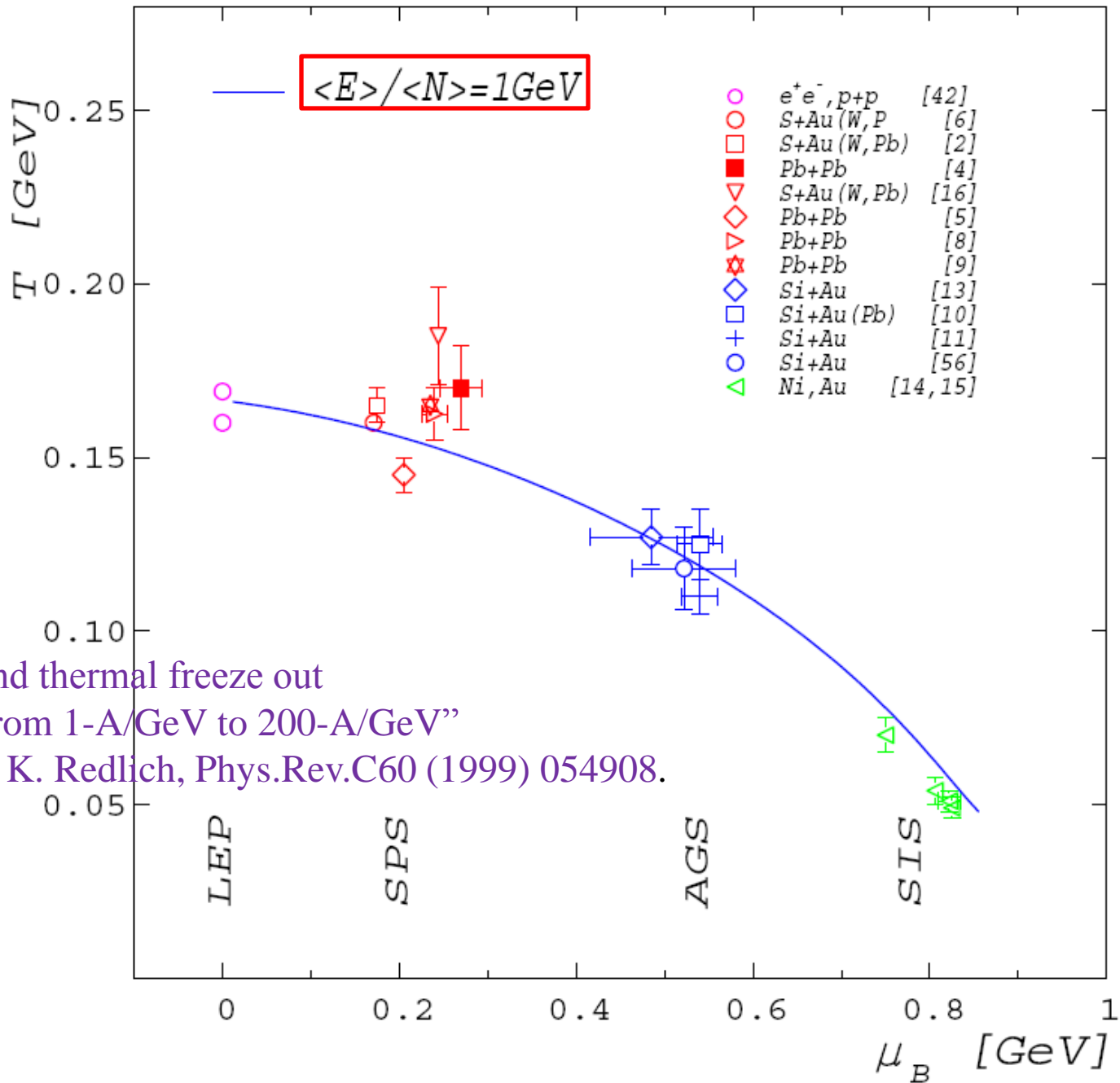
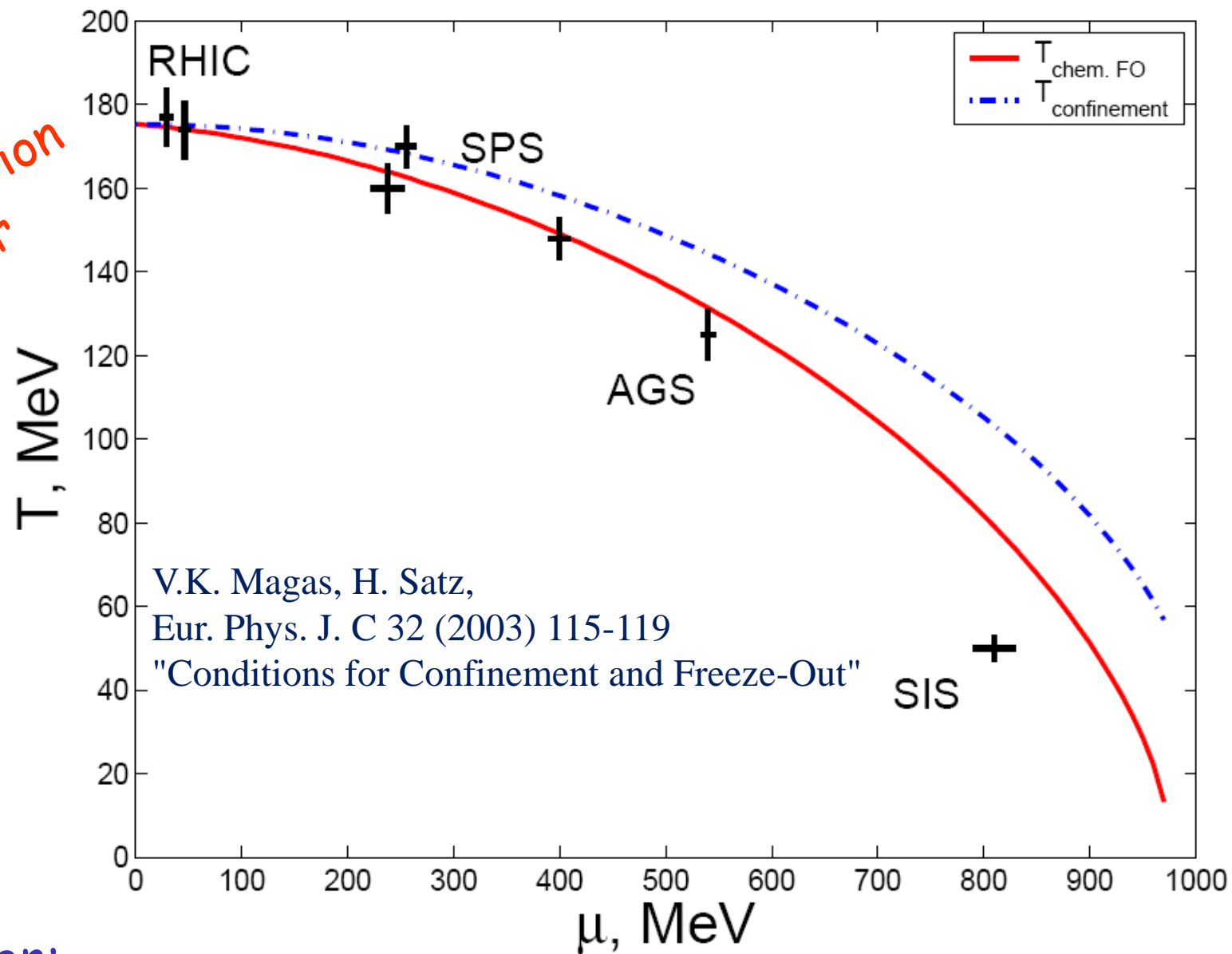


FIG. 1. Chemical freeze-out parameters  $T_{ch}$  and  $\mu_B^{ch}$  obtained at LEP, CERN/SPS, BNL/AGS and GSI/SIS. References to the points are given in table 1. The full line corresponds to an energy density over total particle density of 1 GeV.

The best approximation so far



FO condition:

$$n(T, \mu) = \frac{1.24}{V_h} \left[ 1 - \frac{n_B(T, \mu)}{n(T, \mu)} \right] + \frac{0.34}{V_h} \left[ \frac{n_B(T, \mu)}{n(T, \mu)} \right]$$

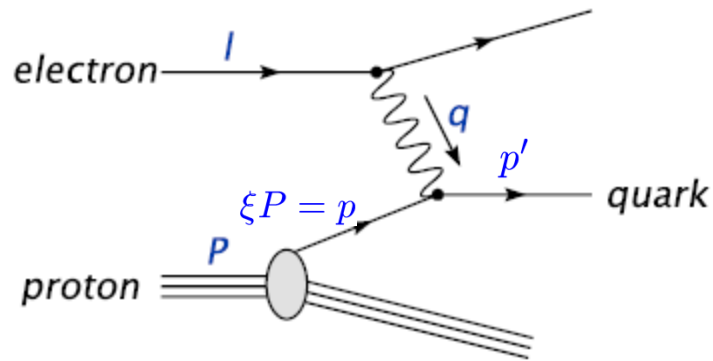
# Color Glass Condensate

Iancu, Leonidov, McLerran, Nucl. Phys. A692, 583 (2001);

Ferreiro, Iancu, Leonidov, McLerran, Nucl. Phys. A710,373 (2002)

Miklos Gyulassy, Larry McLerran, Nucl.Phys.A750,30-63 (2005)

# Deep-inelastic scattering



## Kinetic variables

- momentum transfer  $Q^2 = -q^2$
- Bjorken variable  $x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2P \cdot q}$

for massless quarks

$$0 = p'^2 = (p + q)^2 = 2p \cdot q + q^2 = 2p \cdot q - Q^2$$

$$\Rightarrow 1 = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2\xi P \cdot q} = \frac{x}{\xi}$$

$$\Rightarrow x = \xi$$

Bjorken  $x$  is the momentum fraction of the quark

# Color Glass Condensate

HERA

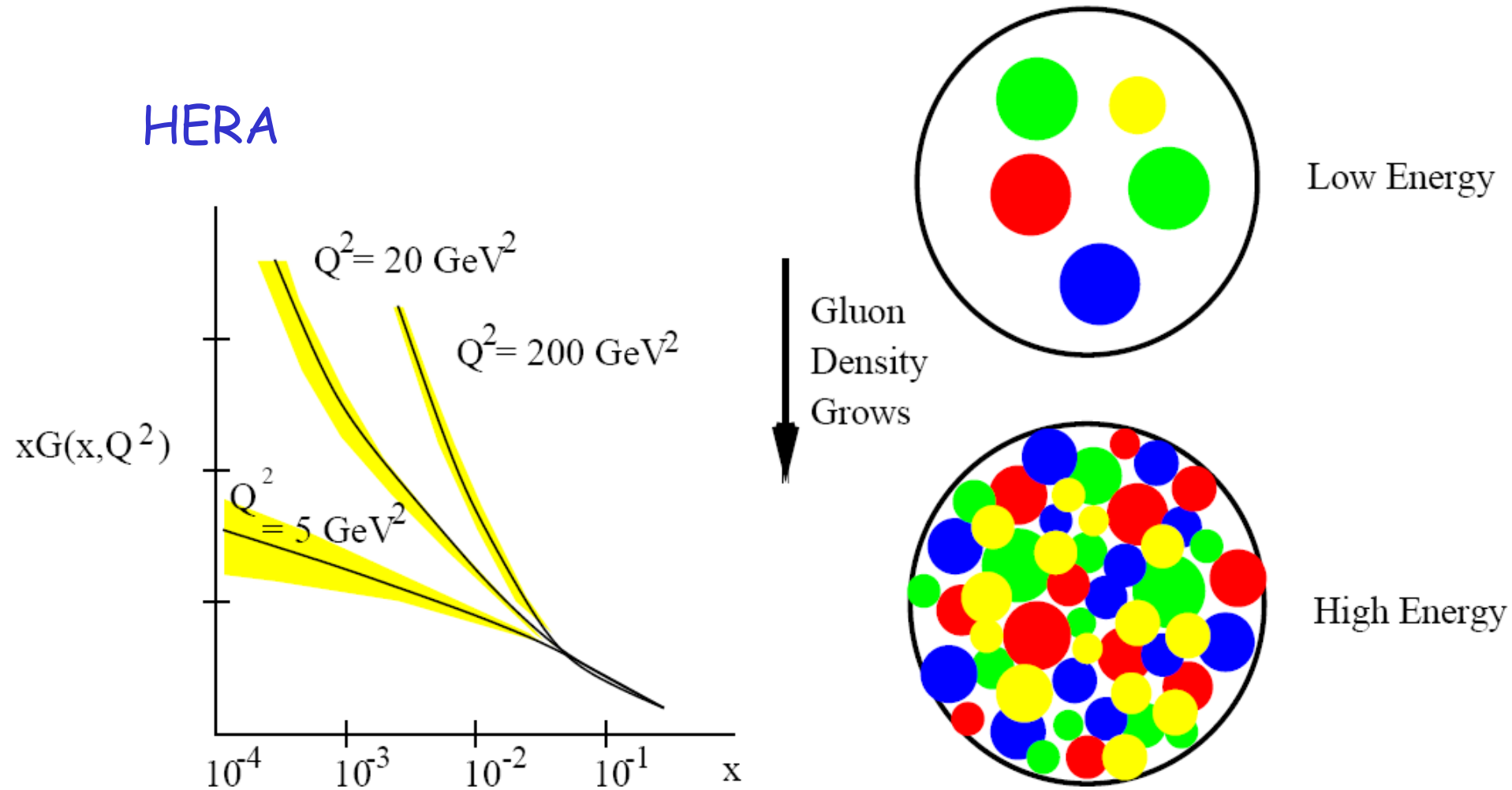
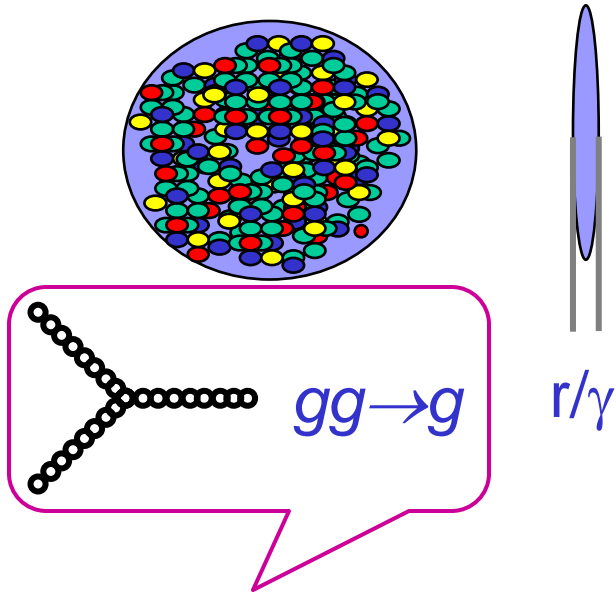


Figure 3: (a) The HERA data for the gluon distribution function as a function of  $x$  for various values of  $Q^2$ . (b) A physical picture of the low  $x$  gluon density inside a hadron as a function of energy

Let us look at the nuclei from the frame moving with large  $\gamma$



Gluons are self-interacting:

probability of the direct reaction  $\sim \rho_g^2$   
probability of the inverse reaction  $\sim \rho_g$

$\rho_g^{max}$  - maximal possible gluon density

However  $\alpha_{QCD} \ll 1$ :

weakly interacting system of gluons

pQCD is applicable

# Let us go to the extreme

$$\gamma \rightarrow \infty \Rightarrow$$

colliding nuclei are 2D sheets

$$\rho_g = \rho_g^{max} - \text{saturation}$$

It is possible to solve  
pQCD in 2D

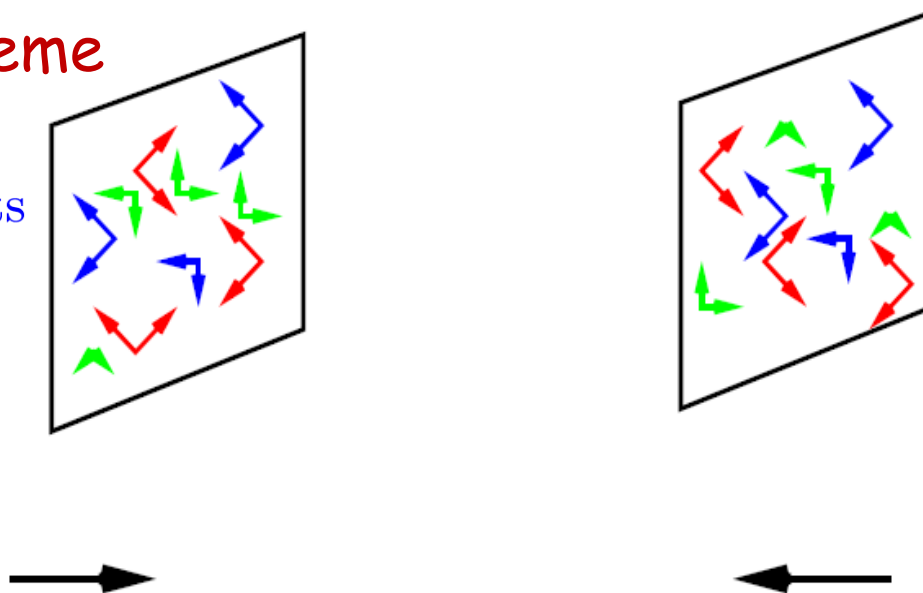
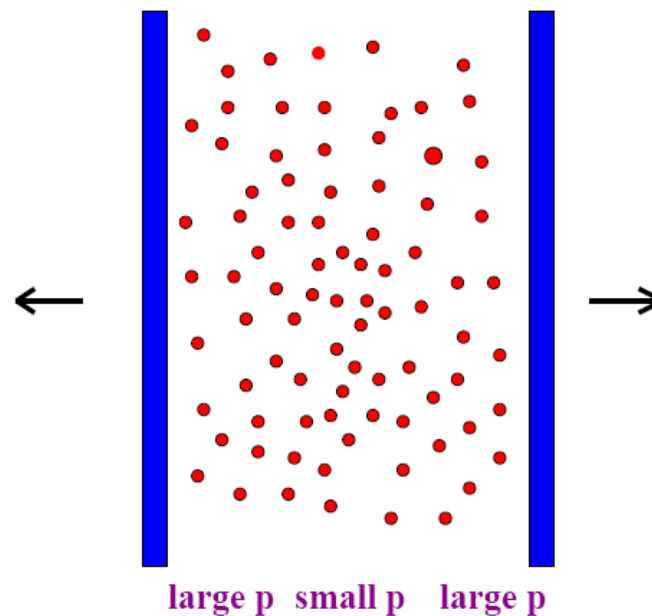


Figure 4: The collision of two sheets of colored glass. The arrows represent the polarization vectors for the gluons which live on the sheets, and their colors correspond to the different colors of gluons



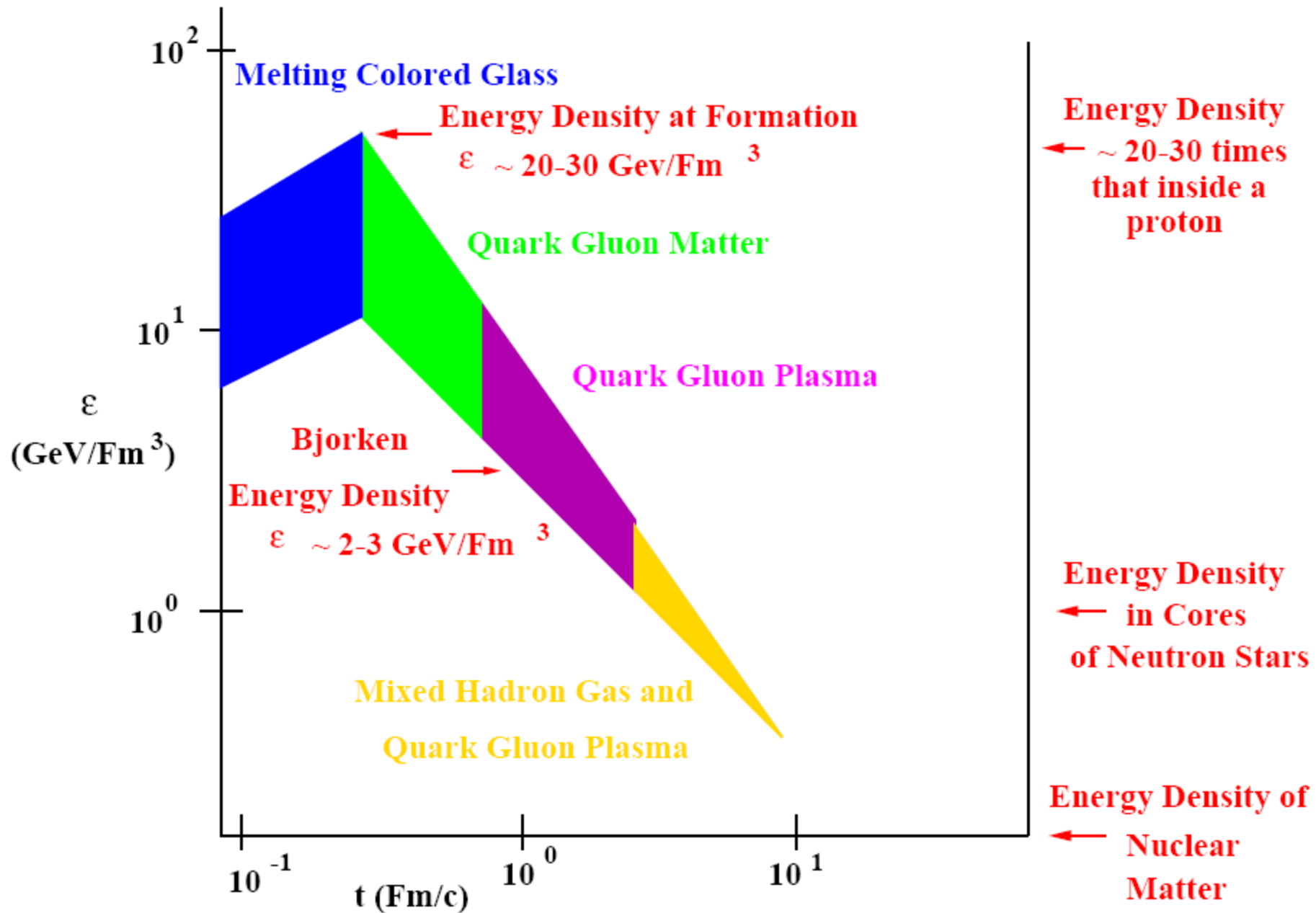


Figure 7: Bounds on the energy density as a function of time in heavy ion collisions.

# Thermal Hadronization and Hawking-Unruh Radiation

# Experimental observation: thermal hadron production

- Hadronic species abundances in high energy  $e^+e^-$ ,  $pp$ ,  $p\bar{p}$ , and AA collisions  
⇒ ideal resonance gas,  $T = 150 - 180$  MeV
- thermal transverse momentum spectra

## What is the thermalization mechanism?

Multiple parton interactions? - AA maybe; but  $e^+e^-$ ,  $pp$ ,  $p\bar{p}$  - no!

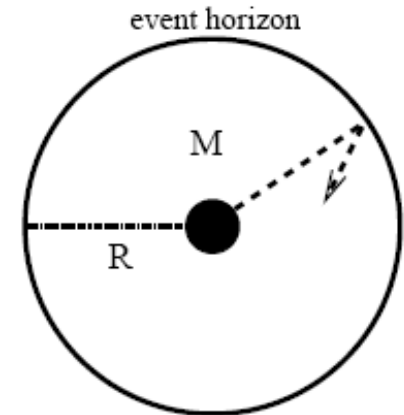
### New idea

Soft hadron production in high energy collisions  
is the Hawking radiation of QCD

# 1. Black Holes and Hawking Radiation

- black hole

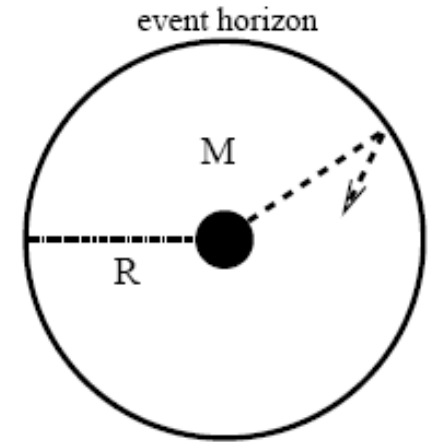
neutron star after gravitational collapse  
large mass  $M$  and compact size  
gravitation so strong that matter &  
light are confined  $\Rightarrow$  event horizon  $R$   
no communication with outside, but...



# 1. Black Holes and Hawking Radiation

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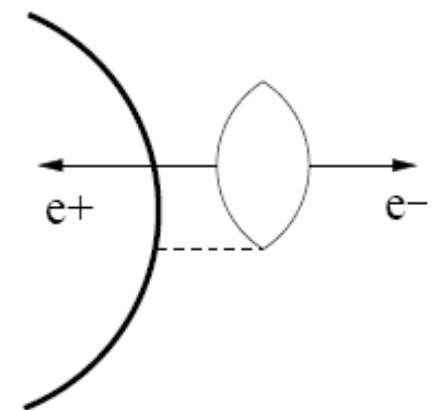
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- Hawking radiation

quantum effect  $\sim$  uncertainty principle  
vacuum fluctuation  $e^+e^-$  outside event horizon, with  $\Delta E \Delta t \sim 1$   
if in  $\Delta t$ ,  $e^+$  falls into black hole,  
then  $e^-$  can escape; equivalent:  
 $e^-$  tunnels through event horizon

(Hawking 1975)



- causality

no information about state of system beyond event horizon  $\Rightarrow$  Hawking radiation must be thermal

$$dN/dk \sim \exp\{-k/T_{BH}\}$$

with black hole temperature  $T_{BH} = \frac{1}{8\pi GM}$

$\Rightarrow$  tunnelling through event horizon  $\rightarrow$  thermal radiation

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- Unruh relation

(Unruh 1976)

stationary observer, mass  $m$  in uniform acceleration  $a$

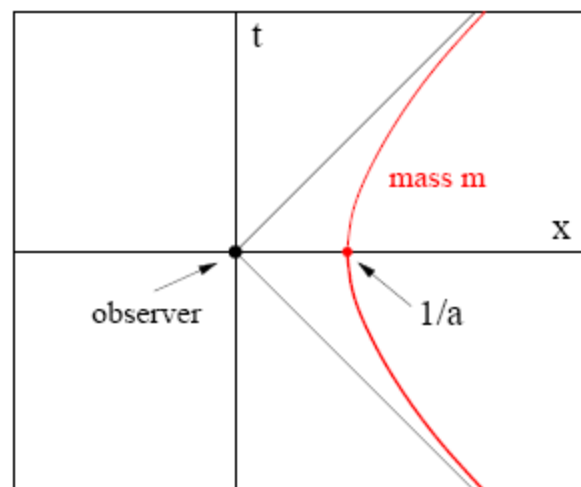
$$\frac{d}{dt} \frac{mv}{\sqrt{1-v^2}} = F$$

with  $v = dx/dt$ ,  $F = ma$ ,  $c = 1$

solution: hyperbolic motion

$$x = \frac{1}{a} \cosh a\tau$$

$$t = \frac{1}{a} \sinh a\tau$$



$1/a$  classical turning point  $\sim$  event horizon  $m$  cannot pass

Only signal:

thermal radiation of temperature

$$T = \frac{a}{2\pi}$$

Unruh: acceleration  $\rightarrow$  event horizon  $\rightarrow$  tunnelling  $\rightarrow$   
thermal radiation

Black hole event horizon (Schwarzschild radius)  $R = 2GM$ :

$$F = ma = G \frac{Mm}{R^2} \Rightarrow a = \frac{GM}{R^2} = \frac{1}{4GM}$$

$$\Rightarrow T_U = \frac{a}{2\pi} = \frac{1}{8\pi GM} = T_{BH}$$

recover Hawking result

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recover Hawking result

In general:

(T. D. Lee 1986, Parikh & Wilczek 2000)

event horizon  $\sim$  information transfer forbidden

$\Rightarrow$  quantum tunnelling  $\sim$  thermal radiation

Relation to QCD?

**Black hole:**

matter and light confined to restricted region of space

**QCD:**

coloured quarks and gluons confined to restricted region of space, colourless from the outside (“white hole”)

Black hole:

matter and light confined to restricted region of space

QCD:

coloured quarks and gluons confined to restricted region of space, colourless from the outside (“white hole”)

Hadrons as black hole analogue in strong interaction physics? (Recami & Castorina 1976, Salam & Strathdee 1978)

Schwarzschild radius of nucleon

$$R_g^n = 2Gm \simeq 1.3 \times 10^{-38} \simeq 10^{-39} \text{fm}$$

Volume of nucleon too big by more than factor  $100^{100}$  to be gravitational black hole

Gravitation  $\rightarrow$  strong interaction:  $Gm^2 \rightarrow \alpha_s$ , so that

$$R_s^n = \frac{2\alpha_s}{m} \simeq 1 \text{ fm}$$

if  $\alpha_s \simeq 2.5$ .

Hadron radius  $\sim$  “strong” Schwarzschild radius

Hadrons  $\sim$  “strong” black (or “white”) holes  
coloured inside, white outside

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More generally:

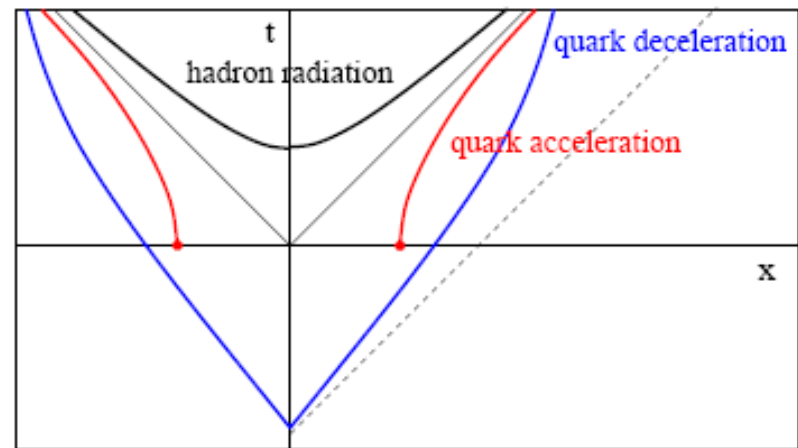
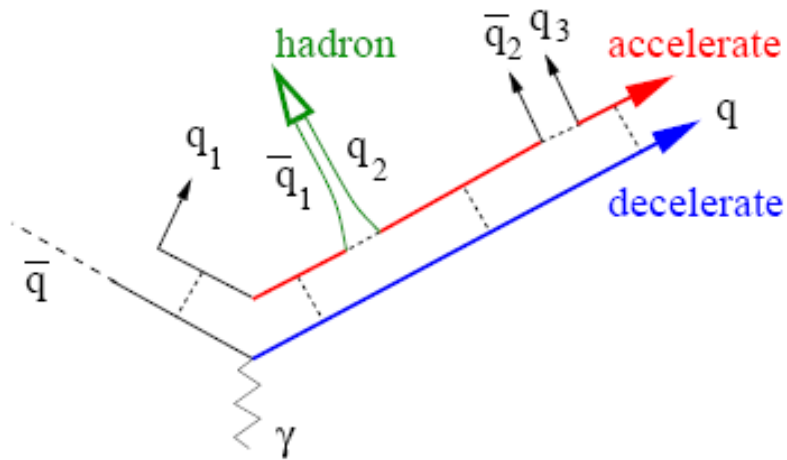
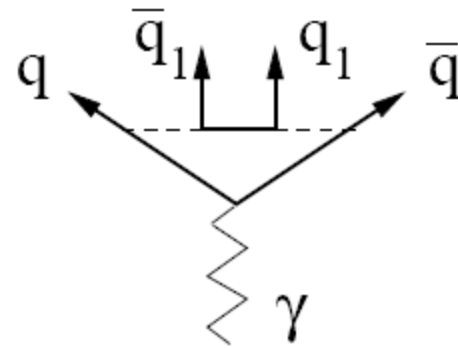
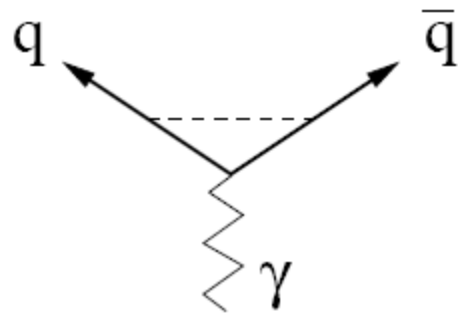
consider interacting hadron medium, quark-gluon plasma  
in the framework of black hole physics concepts

Black hole: event horizon for **all** interactions

White hole: event horizon only for **strong** interactions

# Hadron production in $e^+e^-$ annihilation: “inside-outside cascade”

(Bjorken 1976)



⇒ temperature of hadronic Hawking radiation

$$T_q = \frac{a}{2\pi} \simeq \sqrt{\frac{\sigma}{2\pi}} \simeq 180 \text{ MeV}$$

determines:

species abundances, momentum spectra of emitted hadrons

Viscosity of QGP

Ideal hydrodynamical models give rather good description  
of heavy ion collisions at RHIC

Some theoretical studies indicate that the lowest possible  
viscosity/entropy ration in the matter is

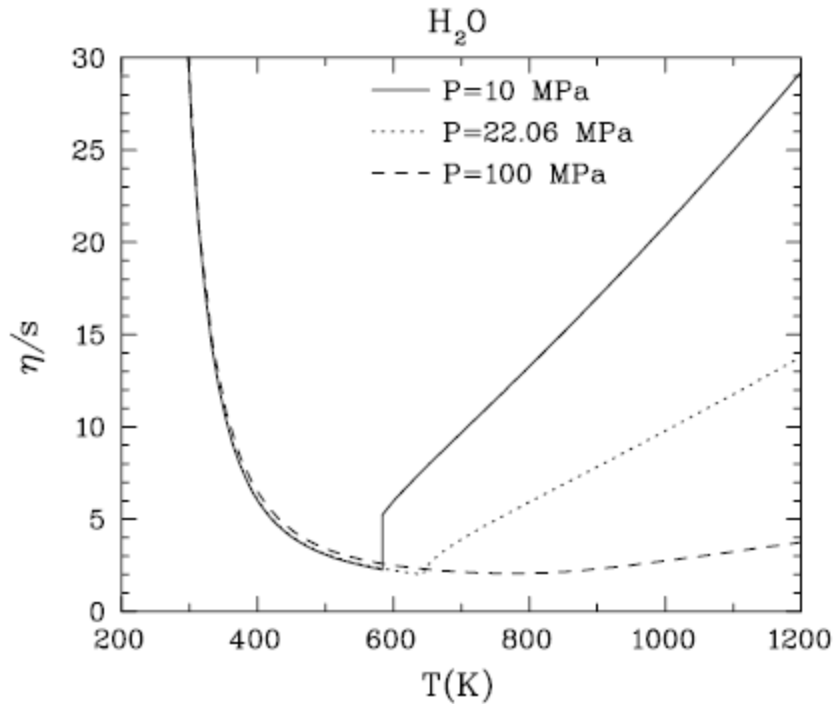
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

⇒ Does this mean that the produced matter has the  
lowest possible viscosity?

## Problems with low-viscosity QGP picture

- 1) Physical picture of CNQ scaling contradicts  
low-viscosity QGP

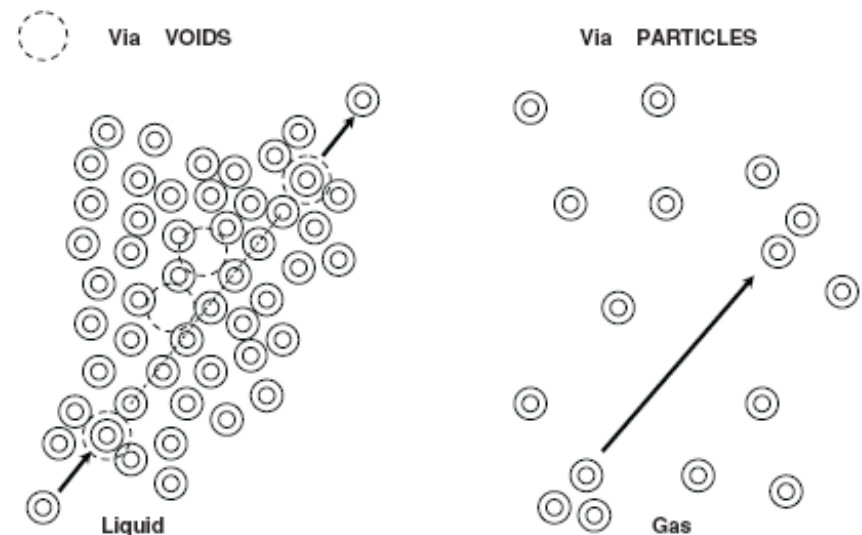
# Viscosity of gas or liquid



Csernai, Kapusta, McLerran,  
Phys. Rev. Lett. 97, 152303 (2006)

**Figure 2.** The ratio  $\eta/s$  for water. The curves correspond to fit the critical pressure. They were constructed using data from NIS at the critical point and slightly above 1.

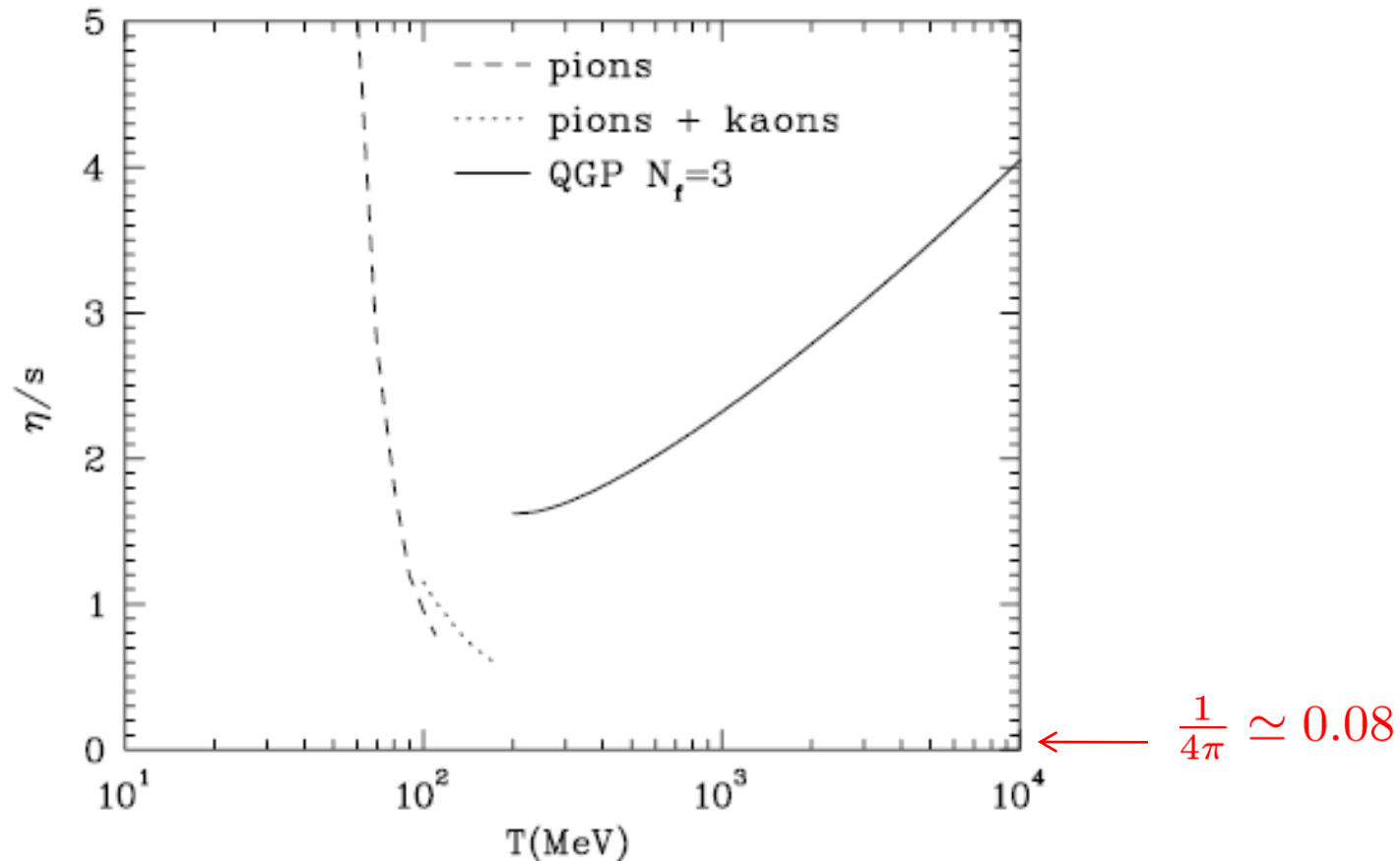
Viscosity – Momentum transfer



## Problems with low-viscosity QGP picture

- 1) Physical picture of CNQ scaling contradicts  
low-viscosity QGP
- 2) Microscopical calculations do not show such a low viscosity

# Hadron matter and QGP viscosity



**Figure 4.** The ratio  $\eta/s$  for the low temperature hadronic phase and for the high temperature quark-gluon phase. Neither calculation is very reliable in the vicinity of the critical or rapid crossover temperature. The value of  $\eta/s$  at the QGP phase transition is estimated to be around one, so it is not very different from other systems at the phase transition point.

## Problems with low-viscosity QGP picture

- 1) Physical picture of CNQ scaling contradicts  
low-viscosity QGP
- 2) Microscopical calculations do not show such a low viscosity  
CNQ flow scaling is a direct measurement, while
- 3) Low-viscosity QGP is a model dependent result

# Viscous effects

“Taken together, hybrid model results show that in this approach one achieves very reasonable, and at the moment probably the best description of the data.”

Voloshin et al., arXiv:0809.2949

...

“Note that these models employ ideal hydrodynamics, once again arguing for very small viscosity of the early QGP stage. Viscose effects in this approach are totally due to the hadronic cascade phase.”

However, hadronic cascades are not doing well with CNQ scaling, HBT radii, etc.

If we use hydro + sharp FO simultaneous with hadronization -

**Should we expect large viscosity of the matter?!**

Natural for the gas of constituent quarks....

# Viscous effects

Flow is very sensitive to the viscosity!

Voloshin et al.,  
arXiv:0809.2949

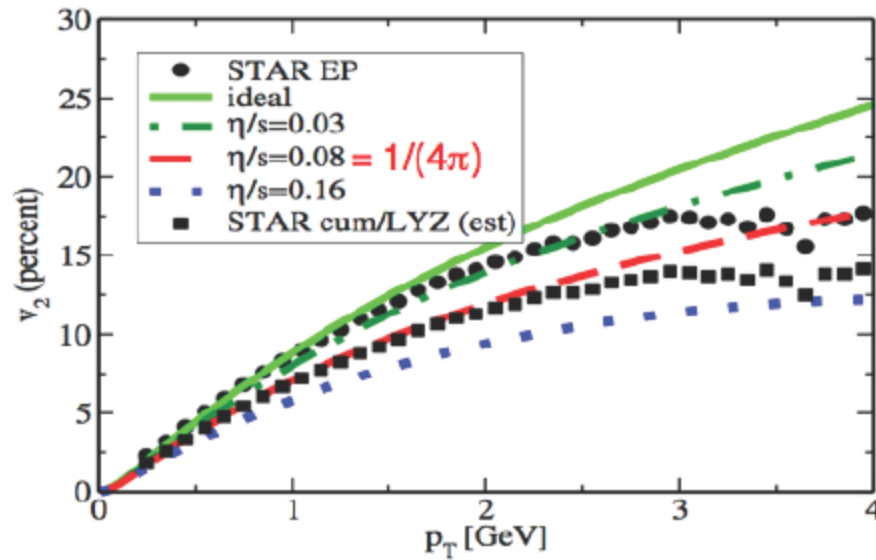


Fig. 24 Viscous hydro calculations [88, 89] compared to minimum bias STAR data [23]. The lower STAR points have an estimated correction for nonflow and fluctuation effects based on the four-particle cumulant and Lee-Yang Zeros methods shown in Fig. 9.

The conclusions should be taken with care -  
[88,89] are 2+1D hydro models based on boost invariance,  
and thus do not conserve global quantities (E,B,S...)

## Problems with low-viscosity QGP picture

- 1) Physical picture of CNQ scaling contradicts  
low-viscosity QGP
- 2) Microscopical calculations do not show such a low viscosity

CNQ flow scaling is a direct measurement, while

- 3) Low-viscosity QGP is a model dependent result

- 4) Relativistic viscous hydrodynamics is not yet a  
well define issue!

All discretized numerical algorithms introduce numerical viscosity, which is difficult to control

# Numerical viscosity

For the Lax method in 1+1D

$$\eta_{num} \sim e^{\frac{\Delta x^2}{\Delta t}}$$

$$\Delta t < \Delta x$$

$$\frac{\eta_{num}}{s} \sim T \frac{\Delta x^2}{\Delta t}$$

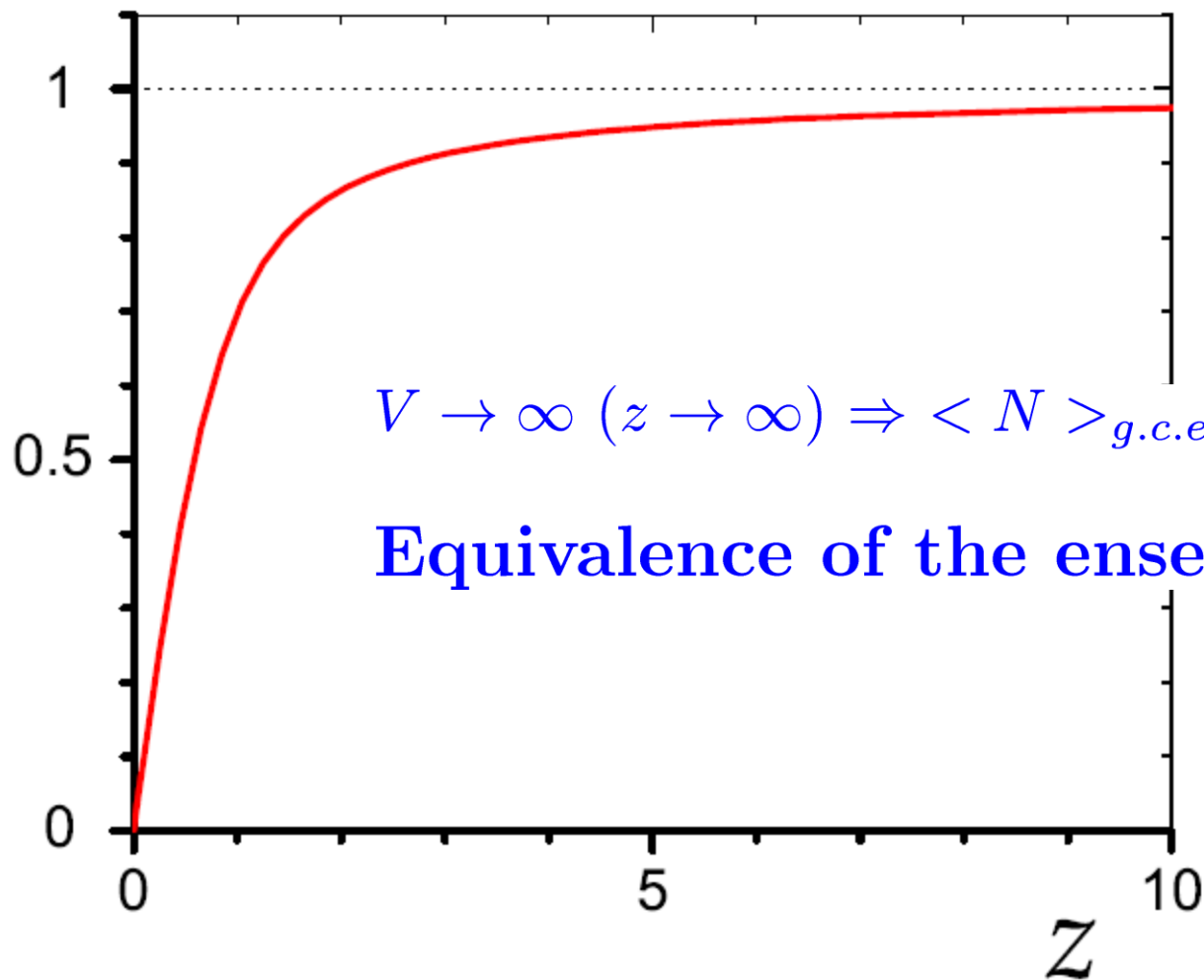
$$T = 200 \text{ MeV}, \Delta x = 1 \text{ fm}, \Delta t = 0.5 \text{ fm}$$

$$\frac{\eta_{num}}{s} \sim 2 = 25 \cdot \frac{1}{4\pi}$$

# Particle Number Fluctuations

Begun, Gazdzicki, Gorenstein, Zozulya  
**Phys.Rev.C70, 034901 (2004)**

$$\frac{\langle N \rangle_{\text{c.e.}}}{\langle N \rangle_{\text{g.c.e.}}}$$



$$V \rightarrow \infty (z \rightarrow \infty) \Rightarrow \langle N \rangle_{\text{g.c.e.}} = \langle N \rangle_{\text{c.e.}}$$

**Equivalence of the ensembles**

$z$  is a single particle partition function

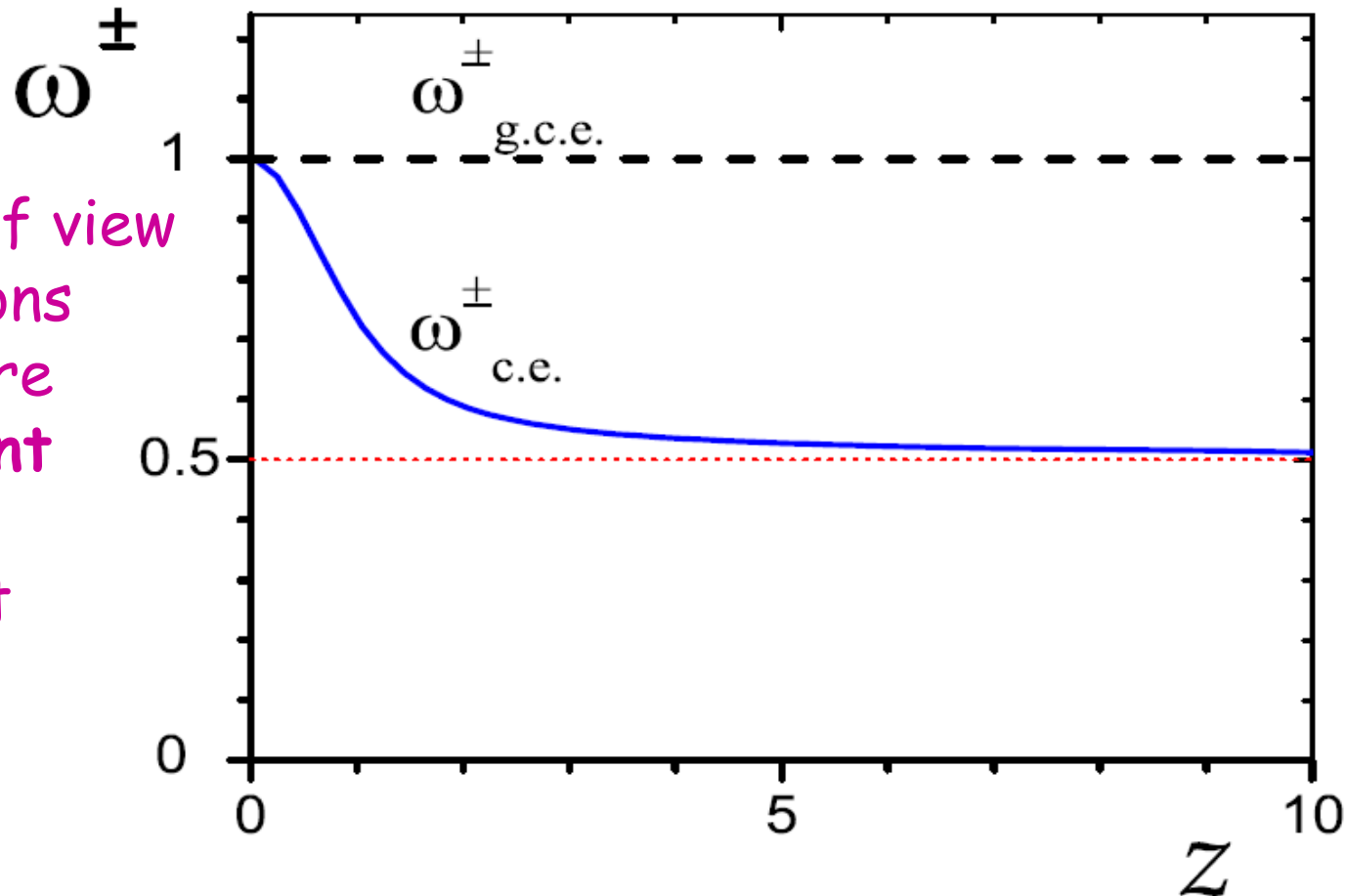
$$z = \frac{V}{2\pi^2} \int_0^\infty k^2 dk \exp \left[ - \frac{(k^2 + m^2)^{1/2}}{T} \right] = \frac{V}{2\pi^2} T m^2 K_2 \left( \frac{m}{T} \right)$$

## Measure of fluctuations - scaled variance

$$\omega^X \equiv \frac{\langle X^2 \rangle - \langle X \rangle^2}{\langle X \rangle}$$

$\omega^X = 1$  for the Poisson distribution

Fluctuations of charged particles (total  $Q=0$ )



From the point of view  
of fluctuations  
ensembles are  
not equivalent  
even in  
 $V \rightarrow \infty$  limit

Thank you!