NEUTRINOLESS DOUBLE BETA DECAY

NUCLEAR STRUCTURE ASPECTS

A. POVES

Departamento de Fisica Teorica and IFT UAM/CSIC, Madrid

Seminario en el Departament d'Estructura i Constituents de la Materia.15 de Octubre de 2008

with E. Caurier (IPHC), J. Menéndez (UAM), and F. Nowacki (IPHC)

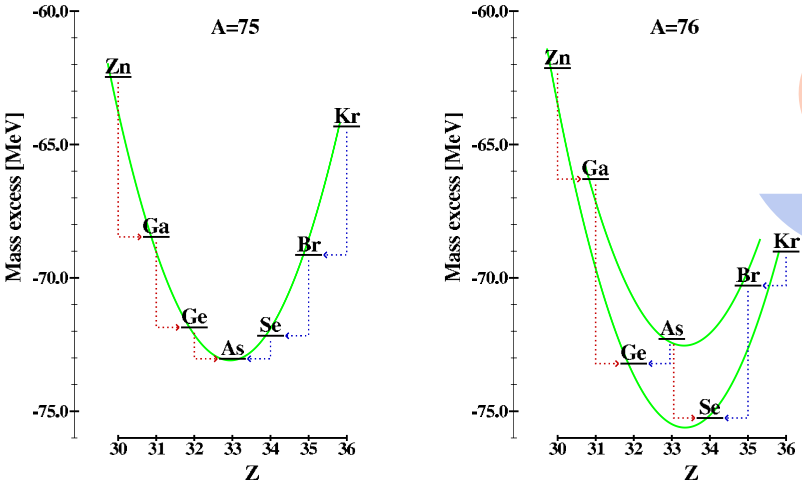
OUTLINE

- Basics.
- The 0ν operators.
- The nuclear wave functions; ISM or QRPA?
- "State of the Art" ISM and QRPA nuclear matrix elements.
- The role of pairing.
- The role of deformation.
- Conclusions.

Some nuclei, otherwise nearly stable, can decay emitting two electrons and two neutrinos ($2\nu\ \beta\beta$) by a second order process mediated by the weak interaction. This decay has been experimentally measured in a few cases.

This process can be observed due to the **nuclear pairing interaction** that favors energetically the even-even nuclei over the odd-odd ones.

INTRO: Double Beta Decay (Isobars $A = 76$)



Double beta decays

When the single beta decay to the intermediate odd-odd nucleus is forbidden, the only decay channel open is the $(2\nu\ \beta\beta)$. For instance, ^{76}Ge decays to ^{76}Se because the decay to ^{76}As is forbidden. The decay probability contains a phase space factor and the square of a nuclear matrix element

$$[T^{2\nu}_{1/2}]^{-1} = G_{2\nu}|M^{2\nu}_{GT}|^2$$

For ^{76}Ge, $T^{2\nu}_{1/2} = (1.3 \pm 0.4)\text{x}10^{21}$ years

$\beta\beta$ emitters with $Q_{\beta\beta}$ >2 Mev

Transition	$Q_{\beta\beta}$ (keV)	Abundance (%) ($^{232}Th = 100$)
$^{110}Pd \rightarrow ^{110}Cd$	2013	12
$^{76}Ge \rightarrow ^{76}Se$	2040	8
$^{124}Sn \rightarrow ^{124}Te$	2288	6
$^{136}Xe \rightarrow ^{136}Ba$	2479	9
$^{130}Te \rightarrow ^{130}Xe$	2533	34
$^{116}Cd \rightarrow ^{116}Sn$	2802	7
$^{82}Se \rightarrow ^{82}Kr$	2995	9
$^{100}Mo \rightarrow ^{100}Ru$	3034	10
$^{96}Zr \rightarrow ^{96}Mo$	3350	3
$^{150}Nd \rightarrow ^{150}Sm$	3667	6
$^{48}Ca \rightarrow ^{48}Ti$	4271	0.2

Double beta decays

If the neutrinos are massive Majorana particles, the double beta decay can also take place without emission of neutrinos ($0\nu\ \beta\beta$).

Has the neutrinoless double beta decay been observed?

There is an unconfirmed claim of discovery by (part of) the Heildelberg-Moscow collaboration (Klapdor 2001, 2004) of the $^{76}Ge \longrightarrow ^{76}Se$ neutrinoless decay with a half-life of (0.7-1.5) x 10^{25} years

The neutrinoless double beta decay

The expression for the neutrinoless beta decay half-life, in the mass mode, for the $0^+ \rightarrow 0^+$ decay, can be brought to the following form:

$$[T_{1/2}^{(0\nu)}(0^+ -> 0^+)]^{-1} = G_{0\nu} \left(M^{(0\nu)} \left(\frac{\langle m_\nu \rangle}{m_e} \right) \right)^2$$

$$M^{(0\nu)} = \left(\frac{g_A}{1.25} \right)^2 \left(M_{GT}^{(0\nu)} - \frac{M_F^{(0\nu)}}{g_A^2} - M_T^{(0\nu)} \right)$$

where $\langle m_\nu \rangle$ is the effective neutrino mass

$$\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k$$

The U's are the matrix elements of the weak mixing matrix. $G_{0\nu}$ is the kinematic phase space factor, and $M^{0\nu}$ the nuclear matrix element (NME) that has Fermi, Gamow-Teller and Tensor contributions.

The Nuclear Matrix Elements

The matrix elements $M^{(0\nu)}_{GT,F,T}$ can be written as,

$$M_K^{(0\nu)} = \langle 0_f^+ | H_K(|\vec{r_1} - \vec{r_2}|)(t_1^- \; t_2^-)\Omega_K | 0_i^+ \rangle$$

$$\text{with} \;\; \Omega_F = 1, \; \Omega_{GT} = \vec{\sigma}_1 \cdot \vec{\sigma}_2, \; \Omega_T = S_{12}$$

$H_K(|\vec{r_1} - \vec{r_2}|)$ are the neutrino potentials ($\sim$1/r) obtained from the neutrino propagator.

The Nuclear Matrix Elements

The neutrino potentials have the following form:

$$H_K^m(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)qdq}{q + E_m - (E_i + E_f)/2}$$

$h_F(q^2) = g_V(q^2)$, and, neglecting higher order terms in the nuclear current, (what we shall not do), $h_{GT}(q^2) = g_A(q^2)$ and $h_T(q^2) = 0$.

The Nuclear Matrix Elements: Closure

The neutrino potentials depend explicitly on the excitation energy of the states of the intermediate nucleus E_m. However, due to the large average energy of the virtual neutrino ($\sim$100 MeV), they can as well be calculated in the closure approximation, that is good to better than 90%.

The Nuclear Matrix Elements

On top of that, other elements have to be taken into account before undertaking the purely nuclear calculations.

- The nucleon finite size is included by means of a dipole form factor
- The Gamow-Teller operator needs to be quenched in the 2ν mode and in the single beta decays. However, this seems not to be an issue in the 0ν decay, because there the purely Gamow-Teller channel plays a minor role.
- The short range correlations, are most often taken into account by means of a Jastrow ansatz, but softer prescriptions are not excluded. Results within the UCOM (Unitary Correlation Operator Method) will be also discussed

How do the 0ν operators act?

The two body transition operators can be written generically as:

$$\hat{M}^{(0\nu)} = \sum_J \left(\sum_{i,j,k,l} M^J_{i,j,k,l} \, \left((a^\dagger_i a^\dagger_j)^J \, (a_k a_l)^J \right)^0 \right),$$

where the indices i, j, k, l, run over the single particle orbits of the spherical nuclear mean field. These operators can be factorized as follows:

How do the 0ν operators act?

$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J^\pi} \;\; \hat{P}_{J^\pi}$$

The operators $\hat{P}_{J^\pi}$ annihilate pairs of neutrons coupled to J^π in the parent nucleus and the operators $\hat{P}^\dagger_{J^\pi}$ substitute them by pairs of protons coupled to the same J^π. The overlap of the resulting state with the ground state of the grand daughter nucleus gives the J^π-contribution to the NME. The –a priori complicated– internal structure of these exchanged pairs is dictated by the double beta decay operators.

The Nuclear Wave Functions

Two main approaches have been traditionally used for the description of the nuclei involved in the transition. The Shell Model with configuration mixing in large valence spaces and the Quasi-particle RPA. To assess the validity of the wave functions, quality indicators are needed such as:

- Good spectroscopy for parent, daughter and grand-daughter, even better if its extend to a full mass region
- GT-strengths and strength functions, 2ν matrix elements, etc.

As we shall surmise later, a better understanding of the structure of the 0ν transition operators **in terms of the main nuclear correlators, pairing and quadrupole,** can be of great help in assessing the accuracy of the nuclear descriptions.

Interacting Shell Model calculations (ISM) *vs* QRPA

- Interaction
  - ISM: Monopole corrected G-matrices
  - QRPA: Realistic or schematic interactions tuned with the g_{ph} and g_{pp} strengths
- Valence space
  - ISM: A limited number of orbits, but all the possible ways of distributing the valence particles among the valence orbits are taken into account.
  - QRPA: A larger number of orbits, but only 1p-1h and 2p-2h excitations from the normal filling are considered (and not all of them)

ISM *vs* QRPA calculations

- Pairing Correlations
  - ISM: Are treated exactly in the valence space. Proton and neutron numbers are exactly conserved. Proton-proton, neutron-neutron, and proton-neutron (isovector and isoscalar) pairing is included
  - QRPA: Only proton-proton and neutron-neutron pairing are considered. They are treated in the BCS approximation. Proton and neutron numbers are not exactly conserved
- Multipole Correlations and Deformation
  - ISM: Are described properly in the laboratory frame. Angular momentum conservation preserved
  - QRPA: The correlations are treated at the RPA level. Permanent deformation is not incorporated

The Valence Spaces

Miscellanea of computationally accessible valence spaces relevant for the description of double beta decay emitters:
(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called *intruder* and the remaining ones are denoted by r_p)

- The *pf* shell; ^{48}Ca
- r_3-$g_{9/2}$: ^{76}Ge, ^{82}Se,
- r_3-$g_{9/2}$ for protons and r_4-$h_{11/2}$ for neutrons; ^{96}Zr, ^{100}Mo
- r_4-$h_{11/2}$ for neutrons and $p_{1/2}$-$g_{9/2}$-r_4 for protons: ^{110}Pd, ^{116}Cd
- r_4-$h_{11/2}$ for neutrons and protons: ^{124}Sn, $^{128-130}$Te, ^{136}Xe

The Strasbourg-Madrid codes can deal with problems involving basis of 10^{11} Slater determinants, using "relatively" modest computational resources

Update of the ISM 0ν results

In the valence spaces r$_3$-$g_{9/2}$ (^{76}Ge, ^{82}Se) and r$_4$-$h_{11/2}$ (^{124}Sn, $^{128-130}$Te, ^{136}Xe) we have obtained high quality effective interactions by carrying out multi-parametrical fits whose starting point is given by realistic G-matrices. In the valence spaces proposed for ^{96}Zr, ^{100}Mo, ^{110}Pd and ^{116}Cd, the results are still to come or subject to further improvement

Update of the ISM 0ν results

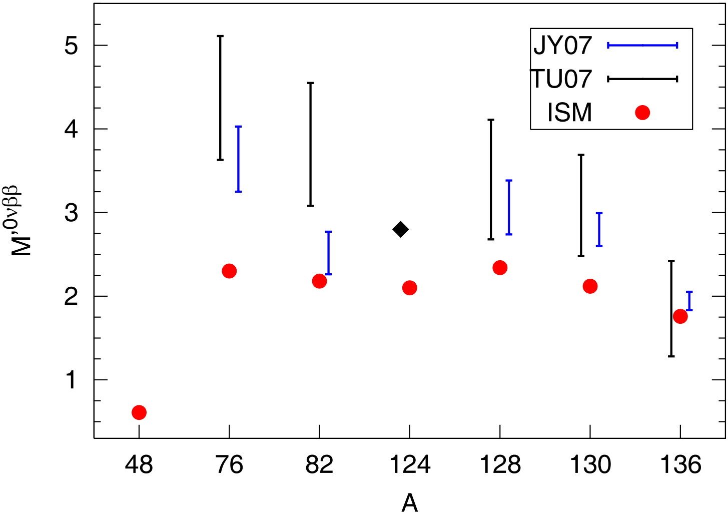
In the present ISM calculations, the Form Factors (FS), the Higher Order Contributions to the Nuclear Current (HOC) and the Short Range Correlations (SRC) are treated in the same way than in the QRPA calculations of the Tubingen and Jyvaskyla groups. The effects of these terms in the NME's, relative to the bare NME's, are very similar in both approaches. This agreement and the fact of having a common treatment of the corrections may contribute to reduce substantially the uncertainties of the NME's

Update of the ISM 0ν results: Jastrow SRC

	$M^{(0\nu)}$(no HOC)	$M^{(0\nu)}$	$\langle m_\nu \rangle$
$^{48}Ca \rightarrow {}^{48}Ti$	0.76	0.59	1.07
$^{76}Ge \rightarrow {}^{76}Se$	2.58	2.22	0.91
$^{82}Se \rightarrow {}^{82}Kr$	2.49	2.11	0.46
$^{124}Sn \rightarrow {}^{124}Te$	2.38	2.02	0.48
$^{128}Te \rightarrow {}^{128}Xe$	2.67	2.26	1.68
$^{130}Te \rightarrow {}^{130}Xe$	2.41	2.04	0.37
$^{136}Xe \rightarrow {}^{136}Ba$	2.00	1.76	0.47

NME's with and without higher order contributions to the nuclear current (HOC)). The effective neutrino mass (in eV) corresponds to $T_{\frac{1}{2}} = 10^{25}$ y. Notice that the Heidelberg-Moscow claim, together with our NME leads to an effective neutrino mass of 1 eV.

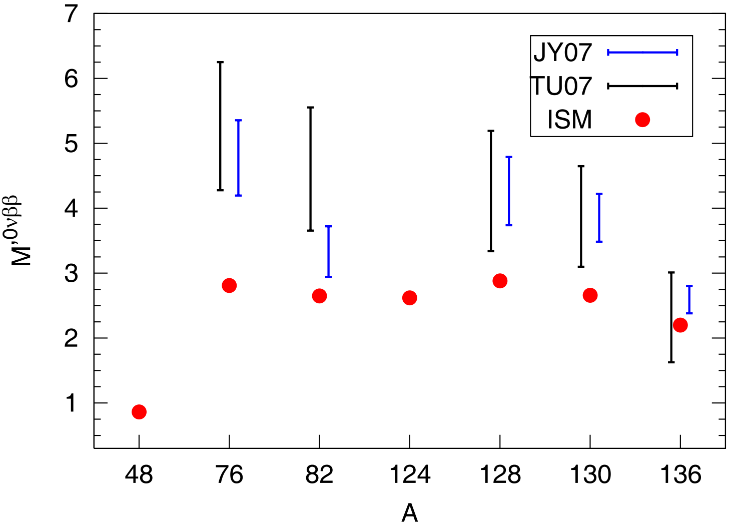
ISM *vs* QRPA NME's ; Jastrow SRC



TU07; QRPA results from Rodin, Simkovic, Faessler, and Vogel 07.

JY07; QRPA results from Kortelainen and Suhonen 07

ISM *vs* QRPA NME's ; UCOM SRC



TU07; QRPA results from Rodin, Simkovic, Faessler, and Vogel 07.

JY07; QRPA results from Kortelainen and Suhonen 07

ISM *vs* QRPA NME's

The range of QRPA values shown in the figures derives from the different choices of g_{pp} and g_A.. The larger values correspond to g_A=1.25 and the smaller ones to g_A=1.0. The ISM numbers are obtained with g_A=1.25.

Both the QRPA and the ISM calculations include the higher order corrections. The short range correlations are modeled by a Jastrow factor or by the UCOM ansatz.

The NME's of the two main QRPA groups are now compatible in most decays, which was not the case less than one year ago. These are good news.

However, except for ^{136}Xe, the ISM NME's are systematically smaller than the QRPA ones. (Preliminary calculations by the Tubingen group of the ^{124}Sn NME give a value 2.8, not very far from the ISM prediction. Are these bad news?

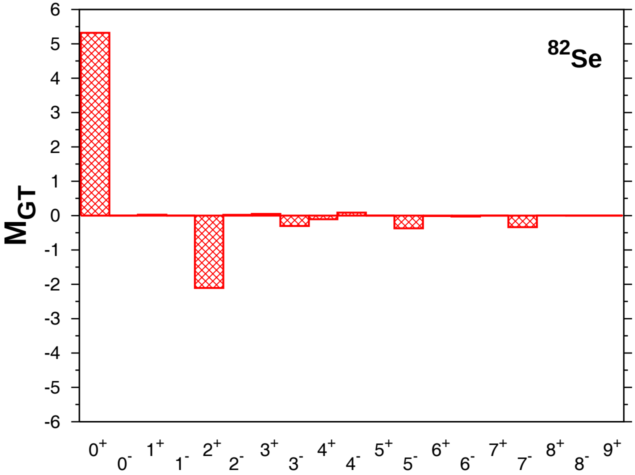
How do the 0ν operators act

Trying to unveil the physics hidden in these results we come back to the factorized operators

$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J^\pi} \ \ \hat{P}_{J^\pi}$$

and decompose the NME in the $^{82}Se \rightarrow {}^{82}Kr$ decay as a sum of the contributions of the pairs with different values of J^π

The contributions to the NME as a function of the J^{π} of the decaying pair: ^{82}Se $\longrightarrow$ ^{82}Kr



Pairing Shows Up

These results are very suggestive, because the leading contribution corresponds to the decay of J=0 pairs, whereas the contributions of the pairs with J>0 are either negligible or have opposite sign to the dominant one.

If we went to the limit of pure pairing correlations, i.e. when the initial and final states have generalized seniority zero, there will be no canceling contributions and therefore the matrix element will be maximal.

Pairing Shows Up

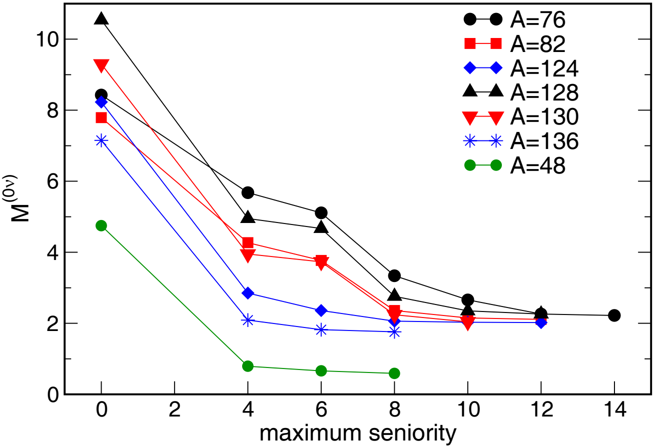
That the J=0 contribution be large can only be understood if the J=0 pairs created or destroyed by the double beta decay operators resemble to the nucleon pairs produced by the nuclear pairing interaction.

This behavior is common to all the cases that we have studied. It also occurs in the QRPA calculations, in whose context it has been previously discussed by Engel, Vogel et al.

Pairing Shows Up

Intriguingly, this reveals that the NME's of the neutrinoless double beta decay depend on the pair content of the nuclear wave functions of parent and grand daughter nuclei. As we shall see, if we force the nuclear wave function to be fully paired, the NME's become very large. The nuclear correlations of multipole type (mainly quadrupole) break pairs and reduce the NME's.

The NME's *vs* the maximum seniority of the WF's



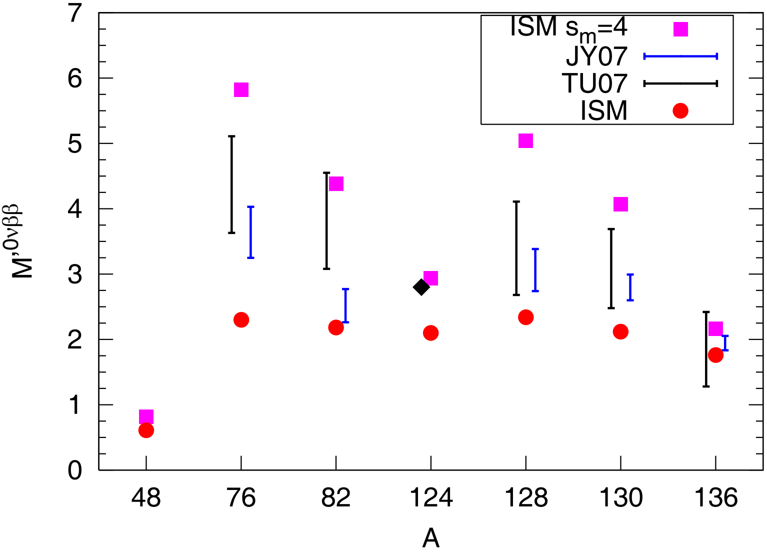
Pay special attention to the s≤4 results because, at leading order, this is the level of ground state correlations in the spherical QRPA calculations based upon a BCS treatment of the pairing interaction

The NME's *vs* the maximum seniority of the WF's

Only when the decaying nucleus is a good superfluid (^{124}Sn), or semi-magic (^{136}Xe), or doubly magic (^{48}Ca), the s$\leq$4 results are close to the exact ones.

And, only in these cases the QRPA NME's are close to the ISM ones!!

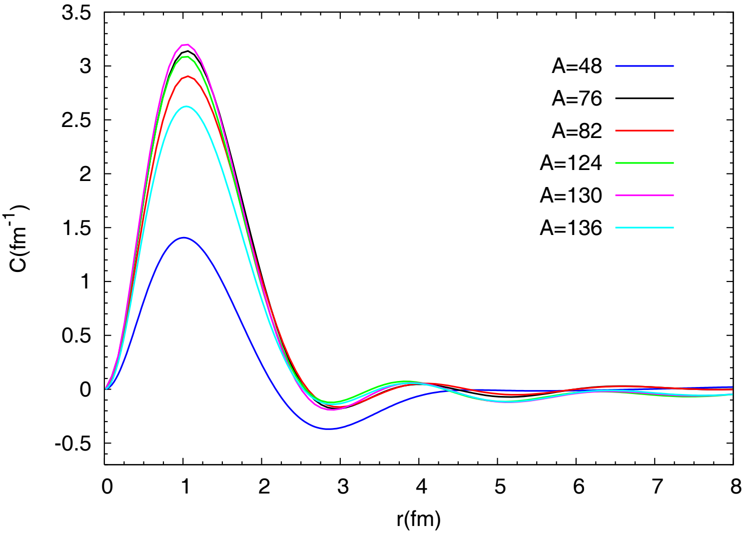
ISM *vs* QRPA NME's ; Jastrow SRC



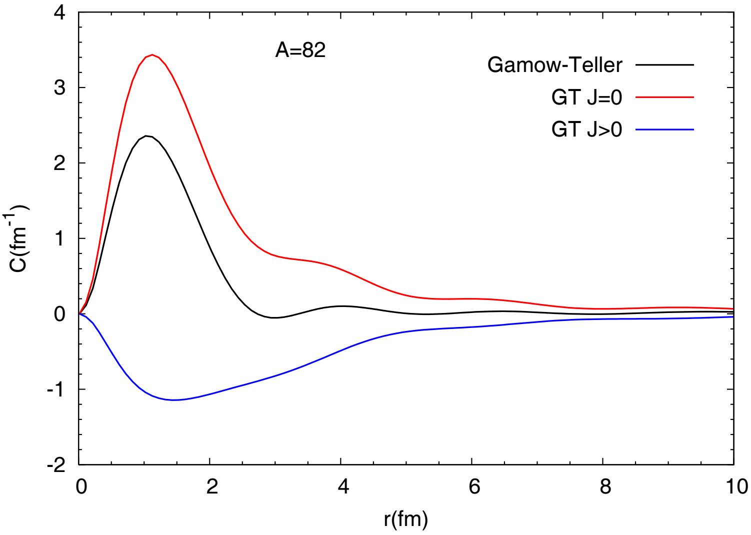
ISM *vs* QRPA NME's

- The QRPA results are reasonably close to the ISM ones at s$\leq$4.
- The ISM values at s$\leq$4 are far from converged, except in the A=48, A=124 and A=136 decays
- Thus, we surmise that, except in these cases, the QRPA overestimates the values of the NME's

The radial structure of the NME's



The radial structure of the NME's

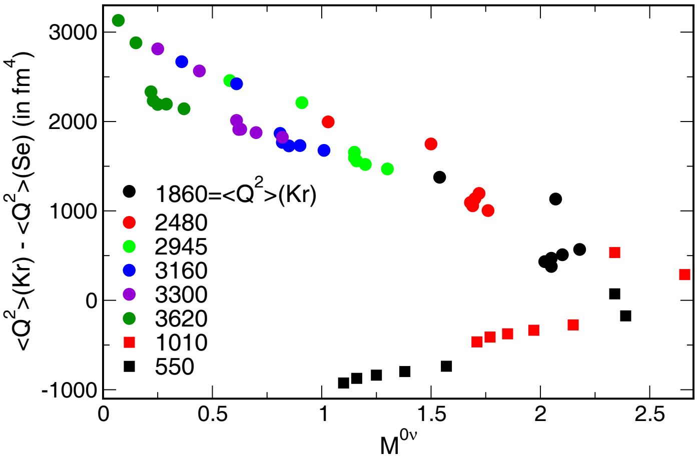


To measure the amount of quadrupole correlations in the ground state of the nuclei that participate in the decay, we refer to the non energy weighted sum rule:

$$\left\langle Q^2 \right\rangle = \sum_i |\left\langle 2_i^+|Q|0^+ \right\rangle|^2$$

Using the ^{82}Kr decay as our test bench, we proceed to compute $\left\langle Q^2 \right\rangle$, first with our standing effective interaction an afterwords with the same interaction to which we add or subtract a quadrupole quadrupole interaction with different strengths ($\lambda\ Q \cdot Q$). And this for many values of λ that need not to be the same for parent and grand daughter.

The role of deformation in the $0\nu\ \beta\beta$ decays



The resulting values of the NME, show a very nice linear trend; the maximum NME is obtained when the value of $\langle Q^2 \rangle$ is the same for both nuclei. As the difference departs from zero, the NME decreases linearly. Work is in progress to understand better this behavior, that can be very relevant for the A=150 decay.

The role of deformation: The ideal and irreal case of a mirror decay

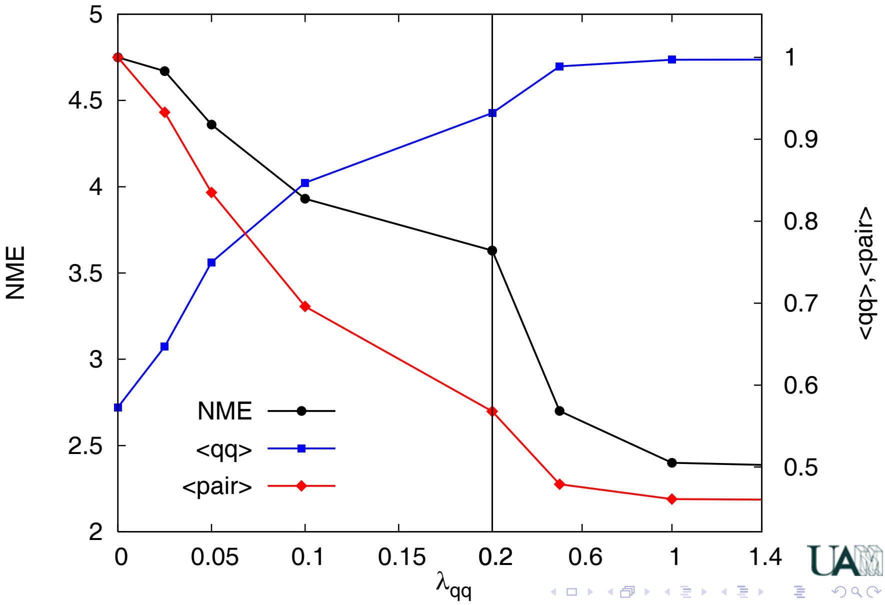
^{50}Cr $\longrightarrow$ ^{50}Fe

^{66}Ge $\longrightarrow$ ^{66}Se

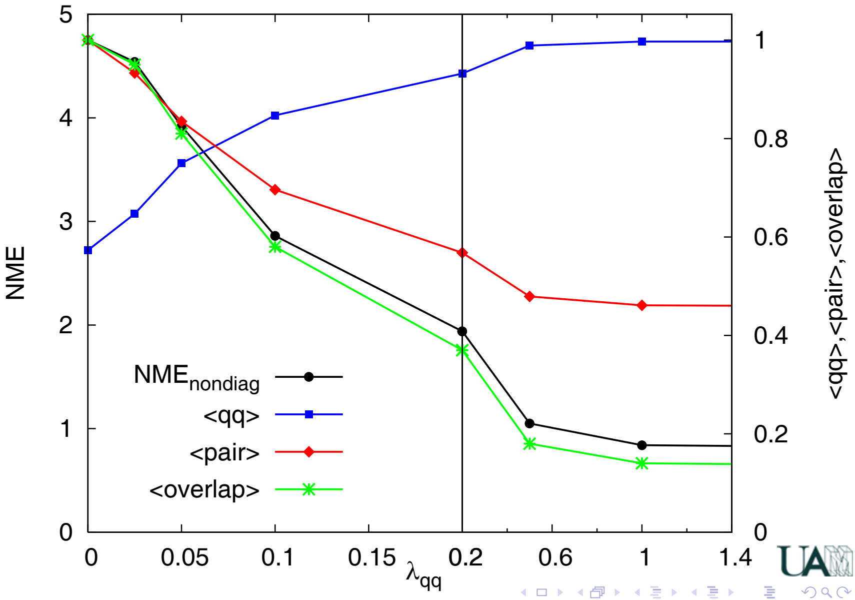
^{110}Xe $\longrightarrow$ ^{110}Ba

In we compute both nuclei with the same interaction, they have the same deformation. If we compute the parent with H_0 and the grand daughter with $H_0 + \lambda Q \cdot Q$ we can evaluate the influence of the *differences* of deformation in the NME

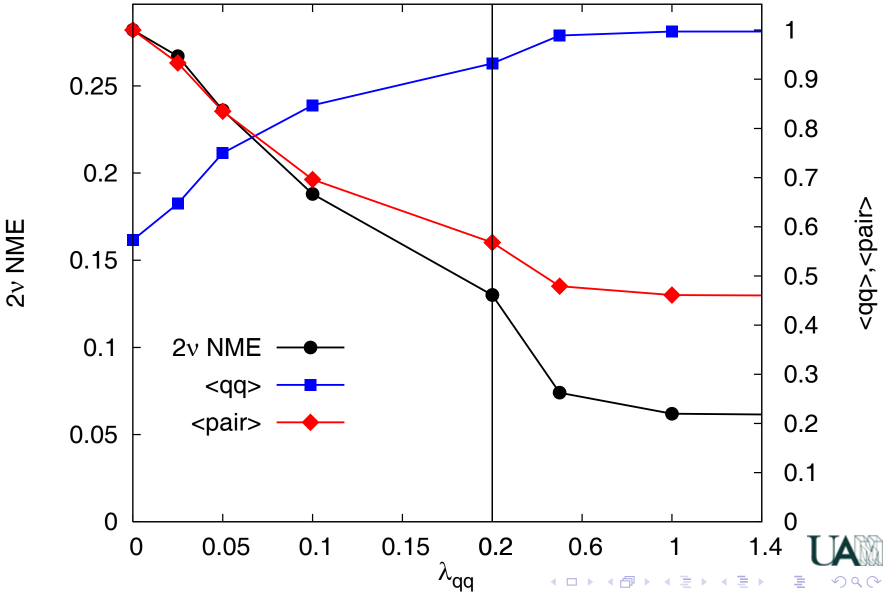
^{66}Ge $\longrightarrow$ ^{66}Se : 0ν : equal deformation



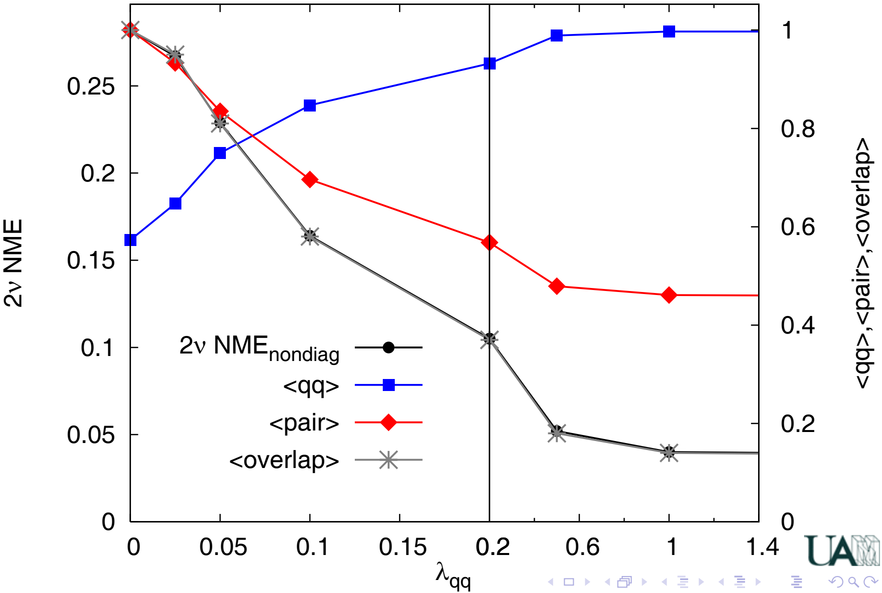
^{66}Ge $\longrightarrow$ ^{66}Se : 0ν : different deformation



$^{66}Ge \longrightarrow {}^{66}Se$: 2ν : equal deformation



^{66}Ge $\longrightarrow$ ^{66}Se : 2ν : different deformation



Conclusions

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters
- We have found that the superfluid correlations in parent and grand daughter favor the neutrinoless decay.
- We have also seen that in the realistic cases, where many other correlations are present, their contributions to the matrix elements come with opposite sign to the the pairing ones.
- In order to take properly into account these cancellations, it is crucial to describe correctly the pair structure of the wave functions.

Conclusions

- State of the art QRPA calculations using the same prescription for the short range correlations are now compatible. Whether to use the Jastrow ansatz or a softer one is still an open question. Nevertheless, the softest possible choice, UCOM, should not increase the NME's by more than 25%
- Low seniority truncations s$\leq$4, similar to those present in the spherical QRPA approaches based in a BCS treatment of the pairing interaction, are shown to fall short in the capture of the proper correlations, and hence to overestimate the nuclear matrix elements in several decays.

Conclusions

- The difference in the amount of quadrupole correlations in the ground states of parent and grand daughter hinders the transition.
- When the quadrupole correlations are large, low seniority approximations fail to treat them properly, and thus, again, the nuclear matrix elements are overestimated.

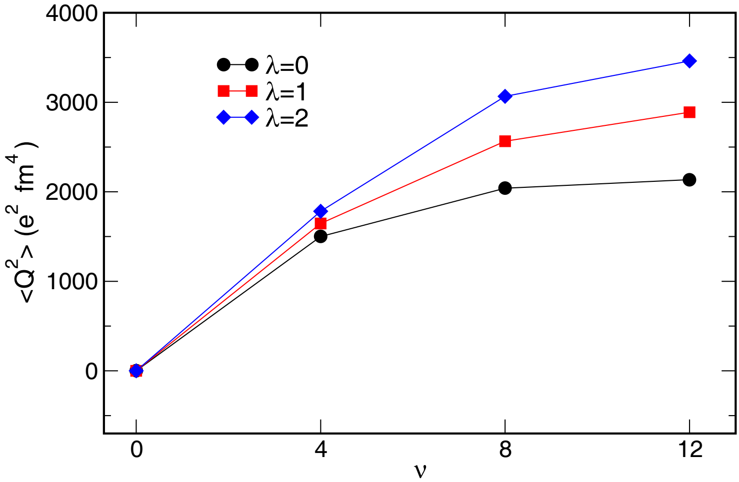
Seniority truncations and quadrupole correlations

To which extent a wave function in the laboratory frame, truncated in seniority, can capture the correlations induced by the quadrupole quadrupole part of the nuclear interaction? To measure the quadrupole correlations of the ground state we refer to the non energy weighted sum rule:

$$\langle Q^2 \rangle = \sum_i |\langle 2_i^+|Q|0^+\rangle|^2$$

Using ^{82}Kr as our test bench, we proceed to compute $\langle Q^2 \rangle$, first with our standing effective interaction and different seniority truncations. We see that at $\nu \leq 4$ (the QRPA reference) some 70% of the full quadrupole correlations are incorporated in the wave function.

Quadrupole correlations in the ground state of ^{82}Kr as a function of the amount of quadrupole-quadrupole interaction ($\lambda\ Q \cdot Q$)



Seniority truncations and quadrupole correlations

How will this behavior evolve when more correlations are enforced in the system? We recalculate the ground state of ^{82}Kr with a new Hamiltonian that consists of the standing one plus a quadrupole quadrupole term ($\lambda\, Q \cdot Q$). The maximum value of $\langle Q^2 \rangle$ in this nucleus and valence space, is $\approx$4500 $e^2 fm^4$.

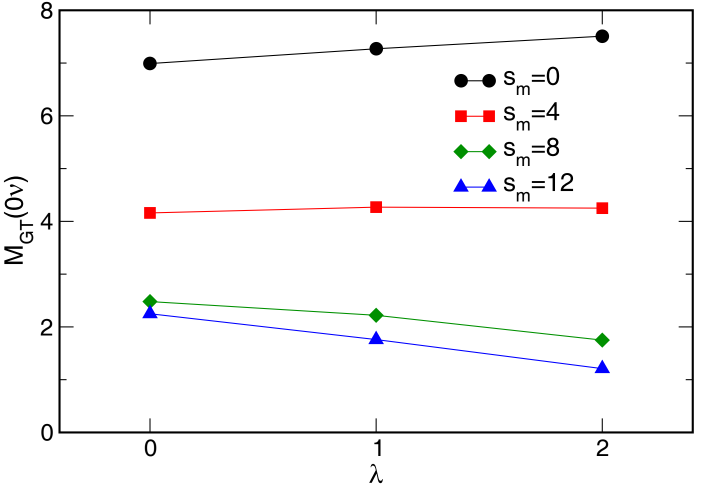
As we try to increase the correlations, the $\nu \leq$4 truncation becomes more and more ineffective. For λ=1, only 57% of the exact correlations are present, and for λ=2 only 50%. The situation is different for ^{82}Se; its values of $\langle Q^2 \rangle$ are independent of λ and similar but a bit smaller than the ^{82}Kr ones for λ=0. Therefore, as we increase λ, the "deformation" of ^{82}Kr grows, whereas that of ^{82}Se remains constant.

The effect of deformation

In fact, by this bias we can explore the effect of the difference of deformation between parent and grand daughter in the NME. For ν=0, we observe that the Gamow Teller matrix element rises as a function of λ. At $\nu \leq 4$, M_{GT} remains constant a function of λ, meaning that, the minor increase of the quadrupole correlations of ^{82}Kr, is barely enough to compensate the increase of M_{GT} at ν=0. On the contrary, the full space results are sensitive to the difference in deformation between parent and grand daughter. The effect goes in the direction of reducing the value of M_{GT}.

In the A=82 decay, doubling the quadrupole correlations in ^{82}Kr, roughly halves M_{GT}. We have studied also the influence of the deformation in a toy model, the 0ν decay of ^{48}Ti to ^{48}Cr reaching the same conclusion; the difference in deformation between the initial and final nuclei hinders the transition

The ^{82}Se $\longrightarrow$ ^{82}Kr 0ν NME as function of the maximum seniority of the wave functions, for different values of the strength of the extra quadrupole-quadrupole interaction



ISM calculations in QRPA-like valence spaces: ^{82}Se

The ISM valence space for the ^{76}Ge and ^{82}Se decays has been traditionally:

$$1p_{\frac{3}{2}},\ 0f_{\frac{5}{2}},\ 1p_{\frac{1}{2}},\ 0g_{\frac{9}{2}}$$

Although only recently full calculations in this space have been possible.

In the QRPA, it is rather:

$$0f_{\frac{7}{2}},\ 1p_{\frac{3}{2}},\ 0f_{\frac{5}{2}},\ 1p_{\frac{1}{2}},\ 0g_{\frac{9}{2}},\ 1d_{\frac{5}{2}},\ 0g_{\frac{7}{2}},\ 2s_{\frac{1}{2}},\ 1d_{\frac{3}{2}}$$

ISM calculations in QRPA-like valence spaces: ^{82}Se

As a first step toward a more complete benchmarking, we have evaluated the influence of the 2p-2h jumps from the $0f_{7/2}$ orbit – ^{56}Ni core excitations– in our results for the ^{82}Se decay. Similar calculations for the ^{76}Ge decay are under way

The calculation in the full r_3g space plus 2p-2h proton excitations from the $0f_{7/2}$ orbit gives a 20% increase of $M^{0\nu}$, but probably we overestimate the amount of core excitations. Our $0f_{7/2}$ proton occupancies, 7.71 and 7.69 in ^{82}Se and ^{82}Kr are smaller than the BCS occupancies of Rodin et al. 7.84 and 7.84. Therefore the above 20% must be taken as an upper bound

The 2ν matrix element remains nearly constant, even if the total Gamow-Teller strengths, (GT+) and (GT-), increase from 0.15 to 0.34 and from 20.5 to 26.9

ISM calculations in QRPA-like valence spaces: ^{136}Xe

We have also computed the ^{136}Xe decay in the r_4h space including 2p-2h excitations from the $0g_{9/2}$ proton orbit and the matrix element increases less than 10%

In another set of calculations, we have included 2p2h neutron excitations toward the $0h_{9/2}$ and $1f_{7/2}$ orbits. The occupancies that we obtain are relatively large (0.25 neutrons in each orbit) and the effect is to increase the matrix element by 15%. It is interesting to note that the increase with the two orbits simultaneously active is equivalent to that obtained including one or another orbit separately. Therefore there is no pile-up of the contributions of the small components of the wave function.

In summary, the ISM results seem to be robust against the inclusion of small components of the wave function