Excitation Modes in Strongly Interacting Bose-Einstein Condensates

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Plan of the talk

- Motivation: Experimental results from Bragg spectroscopy
- Feshbach resonance
- Bogoliubov theory of excitations
- Linear response theory, self energy and the pole structure
- Applications to hard and soft core bosons
- Results: Excitations spectra and dynamic structure function

Bragg spectroscopy

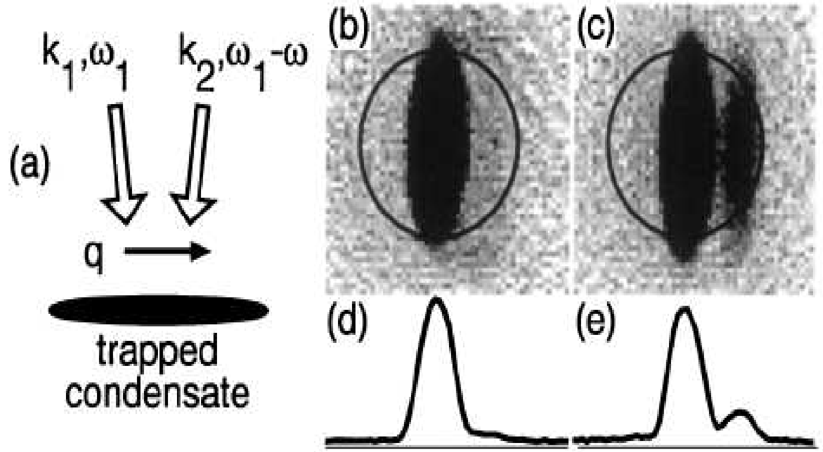

FIG. 1. Observation of momentum transfer by Bragg scattering. (a) Atoms were exposed to laser beams with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and frequency difference ω, imparting momentum $\hbar\mathbf{q}$ along the axis of the trapped condensate. The Bragg scattering response of trapped condensates [(b) and (d)] was much weaker than that of condensates after a 5 ms free expansion [(c) and (e)]. Absorption images [(b) and (c)] after 70 ms time of flight show scattered atoms distinguished from the denser unscattered cloud by their axial displacement. Curves (d) and (e) show radially averaged (vertically in image) profiles of the optical density after subtraction of the thermal distribution. The Bragg scattering velocity is smaller than the speed of sound in the condensate (position indicated by circle). Images are 3.3 × 3.3 mm.

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Excitation of Phonons in a Bose-Einstein Condensate by Light Scattering

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Bragg Spectroscopy of a Strongly Interacting ^{85}Rb Bose-Einstein Condensate

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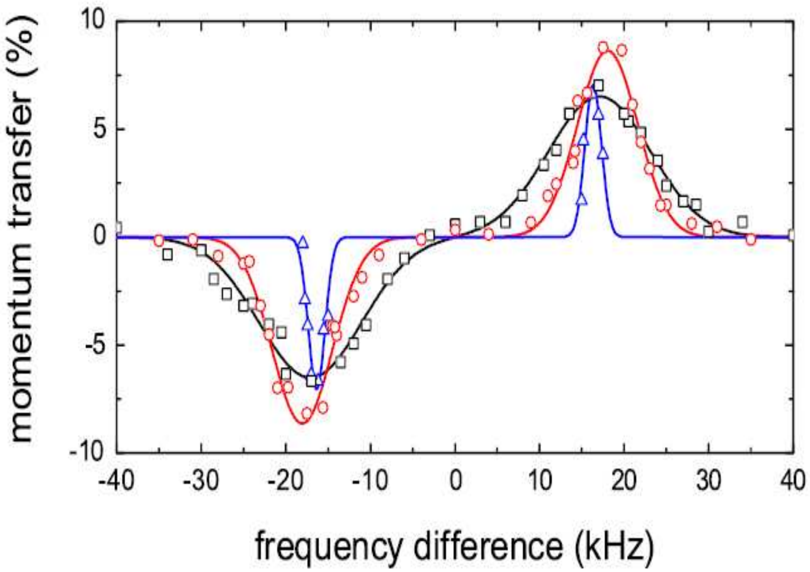
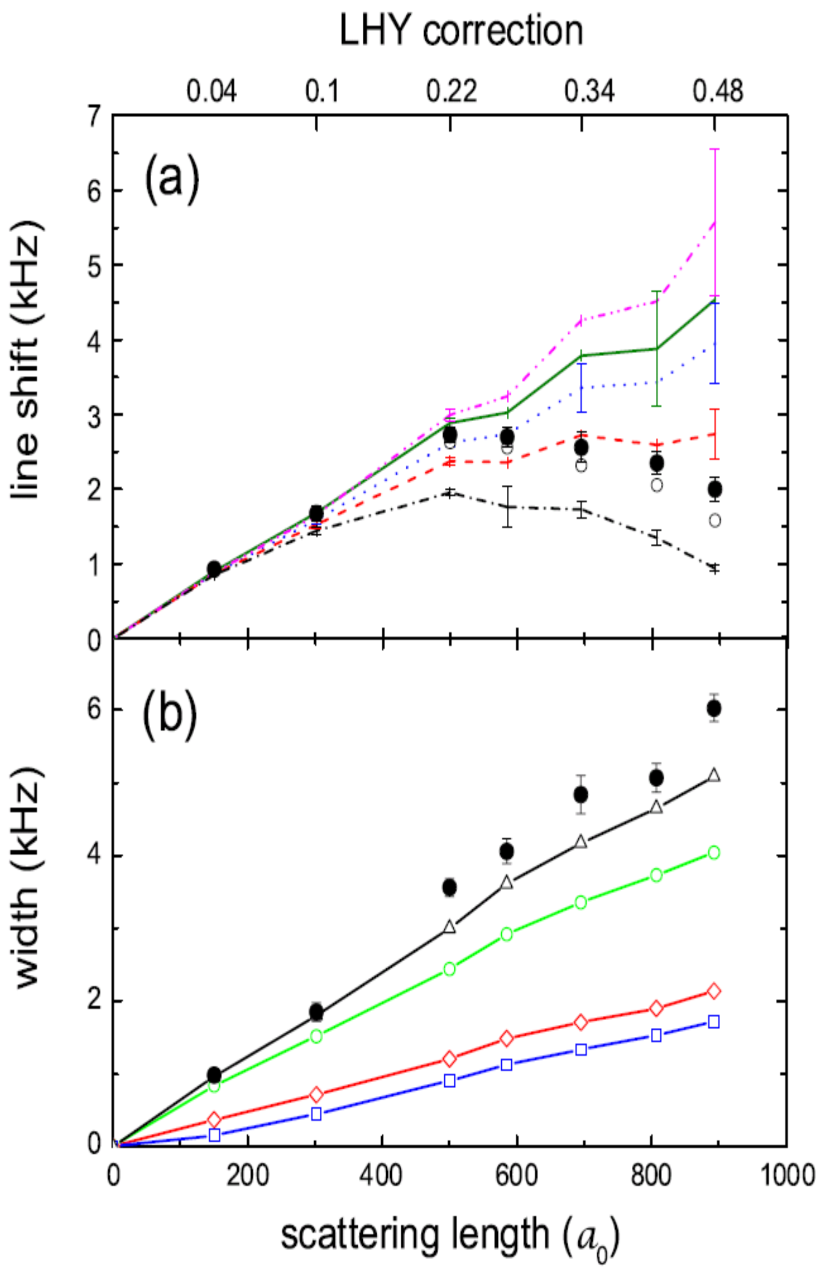


Scattering length is varied by changing the magnetic field

$$a(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

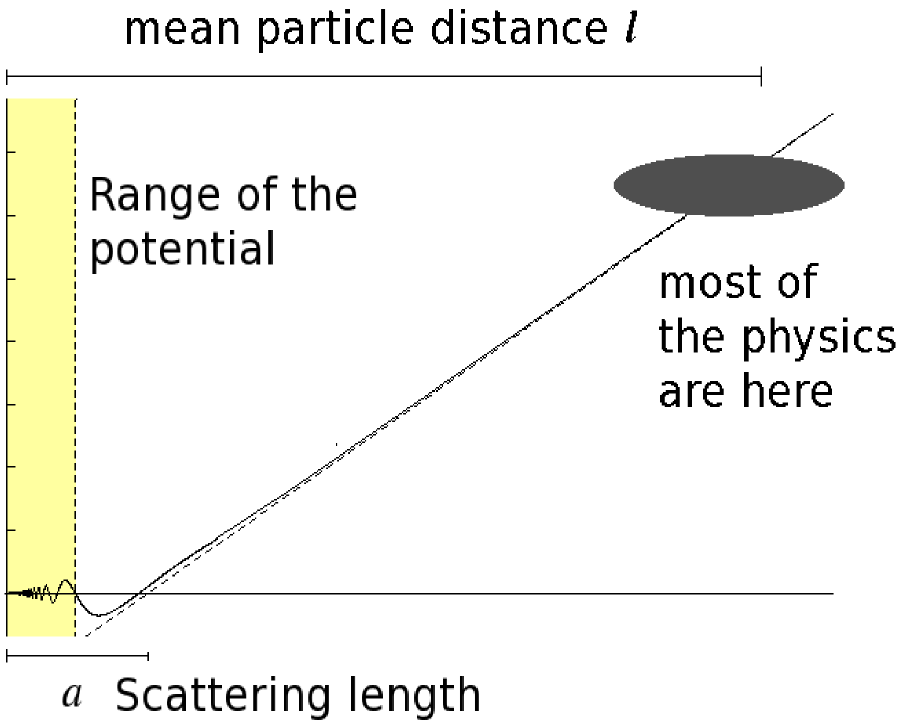
B_0= 155G, ΔB=11G and a_{bg}= -450a_0



Weakly Interacting Systems and Universal Regime

Very dilute systems dominated by two-body correlations

System in gaseous phase



Mean interparticle distance

$$d = \rho^{-1/3} \text{ large}$$

Low energy two-body processes

$$E = \frac{\hbar^2 k^2}{2m} \quad \rightarrow \quad \tan \delta_l(k) \rightarrow k^{2l+1}$$

only s-wave scattering (l=0)

$$\cot \delta_0 = -\frac{1}{ka} + \frac{1}{2} k r_0 + \cdots$$

a = scattering length

only the s-wave scattering length is relevant when $ka \rightarrow 0$

Excitation Spectrum in Bogoliubov Approximation

Bogoliubov model of elementary excitations, valid at low densities when the scattering length dominates

$$\omega(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m}\right)^2 + \frac{8\pi\hbar^2\rho a}{m}\frac{\hbar^2 k^2}{2m}}$$

Expand for low a at fixed momentum k

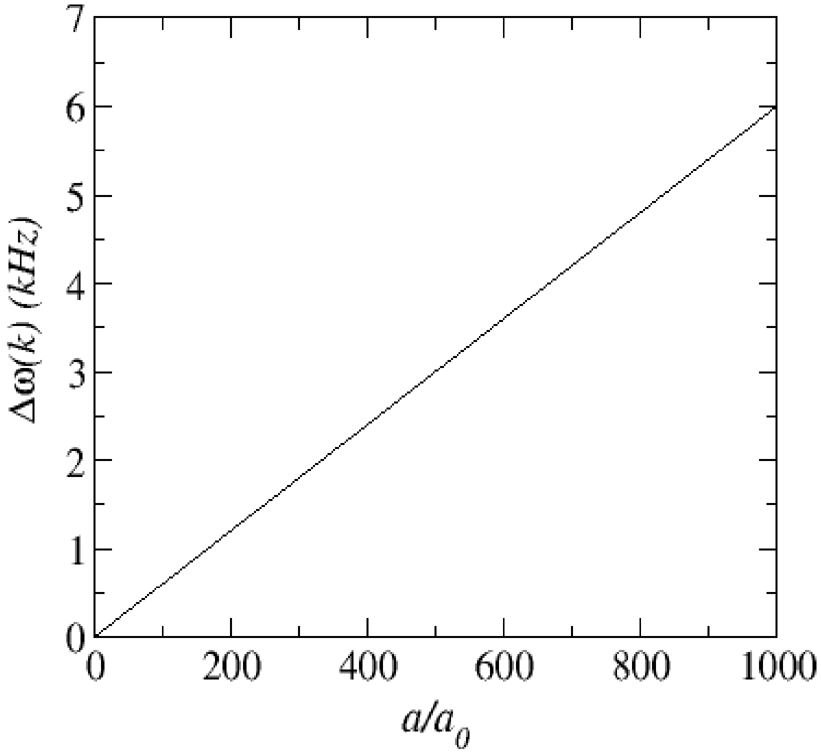
$$\Delta\omega(k) = \omega(k) - \frac{\hbar^2 k^2}{2m} \approx \frac{4\pi\hbar^2\rho}{m} a$$

→ Linear dependence on a

Ex: experimental conditions on a BEC of ^{85}Rb atoms

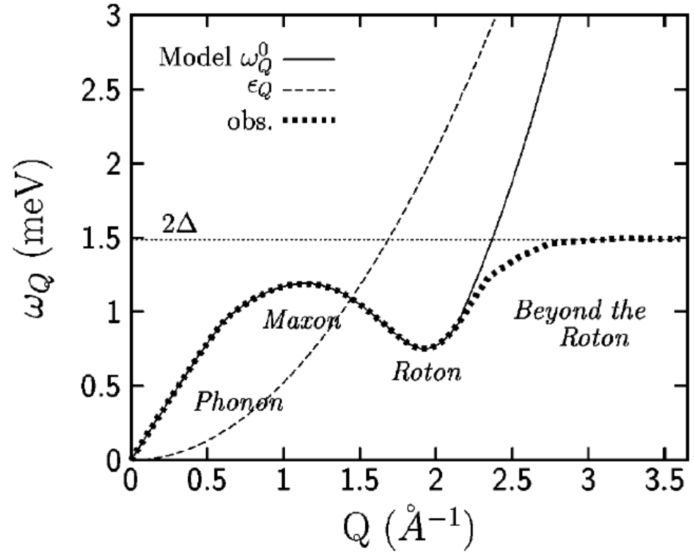
$$\frac{1}{2\pi}\frac{\hbar k^2}{2m} = 15.423\,\mathrm{kHz} \qquad k = \frac{4\pi}{780}\,nm^{-1}$$

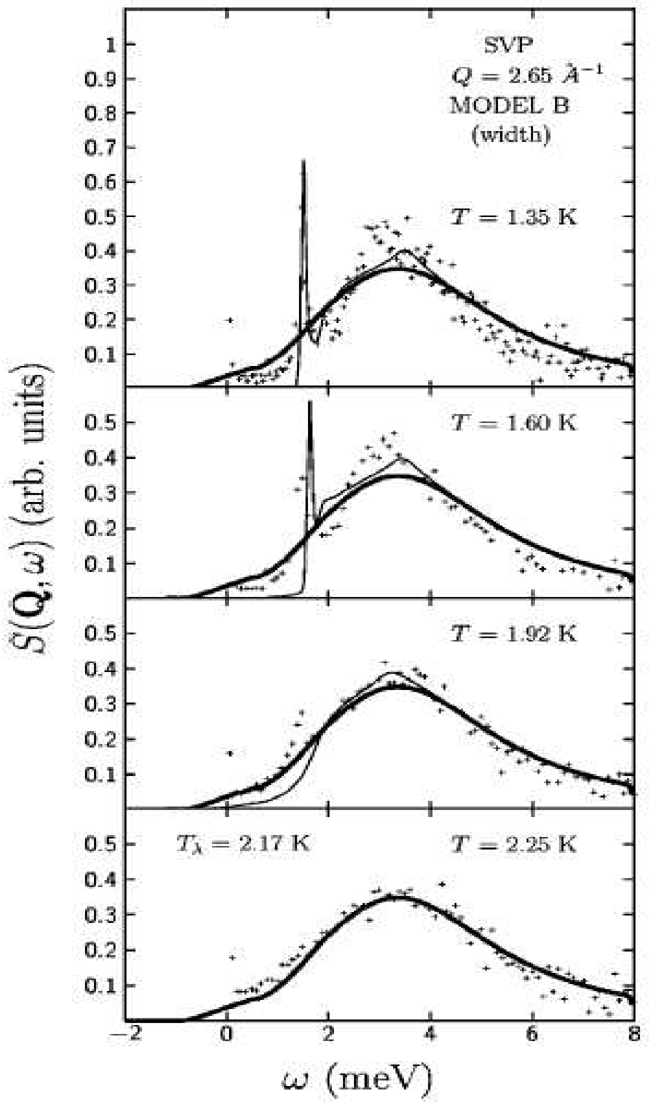
$$\rho = 7.6{\cdot}10^{13}\,cm^{-3}$$



$a_0 = 0.529\,\text{Å}$ Bohr radius

Elementary Excitations in strongly correlated system like liquid ^{4}He





A. R. Sakhel and H. R. Glyde,

Phys. Rev. B 70 144511 (2004)

Dynamic structure function in the linear response

$$\chi(k,\omega) = \frac{\delta\tilde{\rho}_1(k,\omega)}{\rho_0 \tilde{U}_{\mathrm{ext}}(k,\omega)}$$

$$S(k,\omega) = -\frac{1}{\pi}\Im m\left[\chi(k,\omega)\right]$$

$$S(k,\omega) = Z(k)\delta(\hbar\omega - \hbar\omega_0(k)) + S_{\mathrm{mp}}(k,\omega)$$

Self energy in the linear response

$$\chi(k,\omega) = S(k)\left[\frac{1}{\hbar\omega - \varepsilon(k) - \Sigma(k,\omega)} - \frac{1}{\hbar\omega + \varepsilon(k) + \Sigma^*(k,-\omega)}\right]$$

$$\Sigma(k,\omega) = A(k,\omega) - iB(k,\omega) \qquad \Sigma(k,\omega) = \frac{1}{\pi}\int \frac{B(k,\omega')}{\omega-\omega'}d\omega'$$

With the structure function S(k) and Feynman spectrum ε(k).

At T=0 the negative energy contributions vanish. Then

$$S_{\rm mp}(k,\omega) = \frac{1}{\pi}S(k)\frac{B(k,\omega)}{[\hbar\omega - \varepsilon(k) - A(k,\omega)]^2 + [B(k,\omega)]^2}$$

Poles in the linear response

$$\hbar\omega_0(k) = \varepsilon(k) + A(k,\omega_0(k))$$ and $$B(k,\omega_0(k)) = 0$$

$$Z(k) = S(k)\left[1 - \left.\frac{dA(k,\omega)}{d(\hbar\omega)}\right|_{\omega=\omega_0}\right]^{-1}$$

Sum rules

$$
\begin{aligned}
\lim_{k\to 0}\int_0^\infty d\omega\, \frac{S(k,\omega)}{\omega} &= \frac{1}{2mc^2} \\
\int_0^\infty d(\hbar\omega) S(k,\omega) &= S(k) \\
\int_0^\infty d(\hbar\omega)\ \hbar\omega\ S(k,\omega) &= \frac{\hbar^2 k^2}{2m} \\
\int_0^\infty d(\hbar\omega)\ (\hbar\omega)^2\ S(k,\omega) &\to \frac{4}{3}\frac{\hbar^2 k^2}{2m}\langle E_{\mathrm{kin}}\rangle \quad \text{when } k \to \infty
\end{aligned}
$$

where c is the speed of sound, S(k) the static structure function and E_{kin} the kinetic energy.

From the second moment sum rule we get

$$
\lim_{k\to\infty} \Sigma(k,\omega=0) = -\frac{4}{3}\langle E_{\mathrm{kin}}\rangle
$$

Self energy in the uniform limit approximation or in the CBF-form

C. C. Chang and C. E. Campbell, Phys. Rev. B 13, 3779 (1976)

$$\Sigma(k,\omega) = \frac{1}{2}\int \frac{d^3p\, d^3q}{(2\pi)^3\rho_0}\delta(\mathbf{k}+\mathbf{p}+\mathbf{q})\frac{|V_3(\mathbf{k};\mathbf{p},\mathbf{q})|^2}{\hbar\omega - \varepsilon_{\mathrm{F}}(p) - \varepsilon_{\mathrm{F}}(q)}$$

$$V_3(\mathbf{k};\mathbf{p},\mathbf{q}) = \frac{\hbar^2}{2m}\sqrt{\frac{S(p)S(q)}{S(k)}}[\mathbf{k}\cdot\mathbf{p}\tilde{X}(p) + \mathbf{k}\cdot\mathbf{q}\tilde{X}(q) - k^2\tilde{u}_3(\mathbf{k},\mathbf{p},\mathbf{q})]$$

where X(p) is the direct correlation function, N(q) is the nodal sum and u3(k,p,q) the triplet correlation function.

Structure function

Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\nabla_j^2 + \sum_{1=i<j}^{N} V(r_{ij})$$

Two-particle interaction

$$V(r) = \begin{cases} V_0, & r < R \\ 0 & r > R \end{cases}$$

Wave function

$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \prod_{1=i<j}^{N} f(r_{ij})$$

Euler equation

$$\frac{\delta\langle\Psi_0|H|\Psi_0\rangle}{\delta f(r)} = 0$$

Structure function

$$S(k) = \frac{k}{\sqrt{k^2 + \frac{4m}{\hbar^2}\tilde{V}_{\mathrm{p-h}}(k)}}$$

Effective interaction

$$V_{p-h}(r) = g(r)V(r) + [g(r) - 1]\, w_{\mathrm{ind}}(r) + \frac{\hbar^2}{m}\left(\nabla\sqrt{g(r)}\right)^2$$

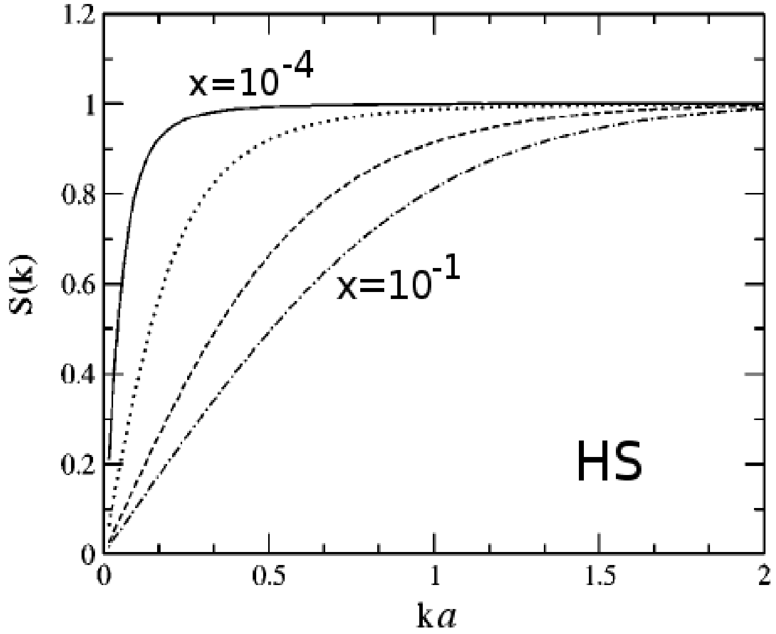
$$\tilde{w}_{ind}(k) = -\frac{\hbar^2 k^2}{4m}(2S(k) + 1)\left(1 - \frac{1}{S(k)}\right)^2$$

Scattering length

$$a = R[1 - \tanh(K_0 R)/(K_0 R)]$$

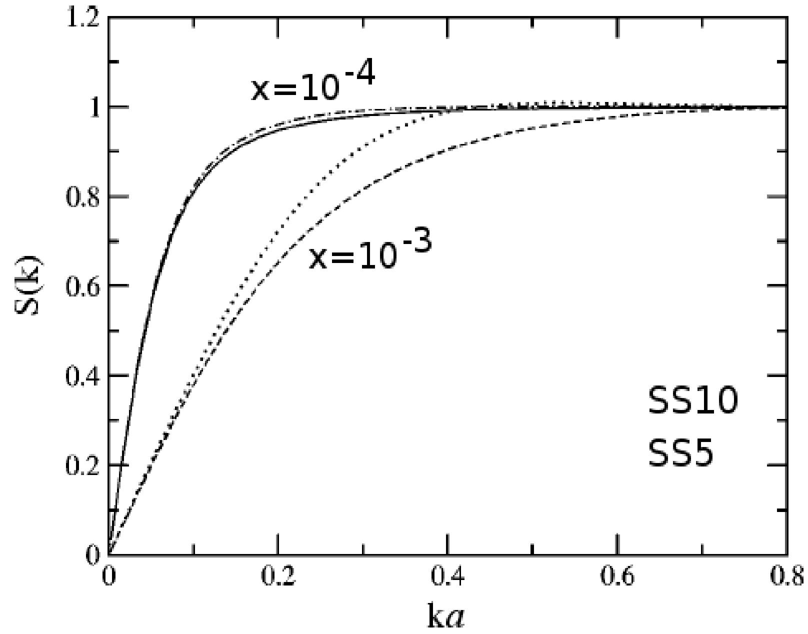
$$K_0^2 = 2mV_0/\hbar^2$$

HNC/EL Results for S(k)



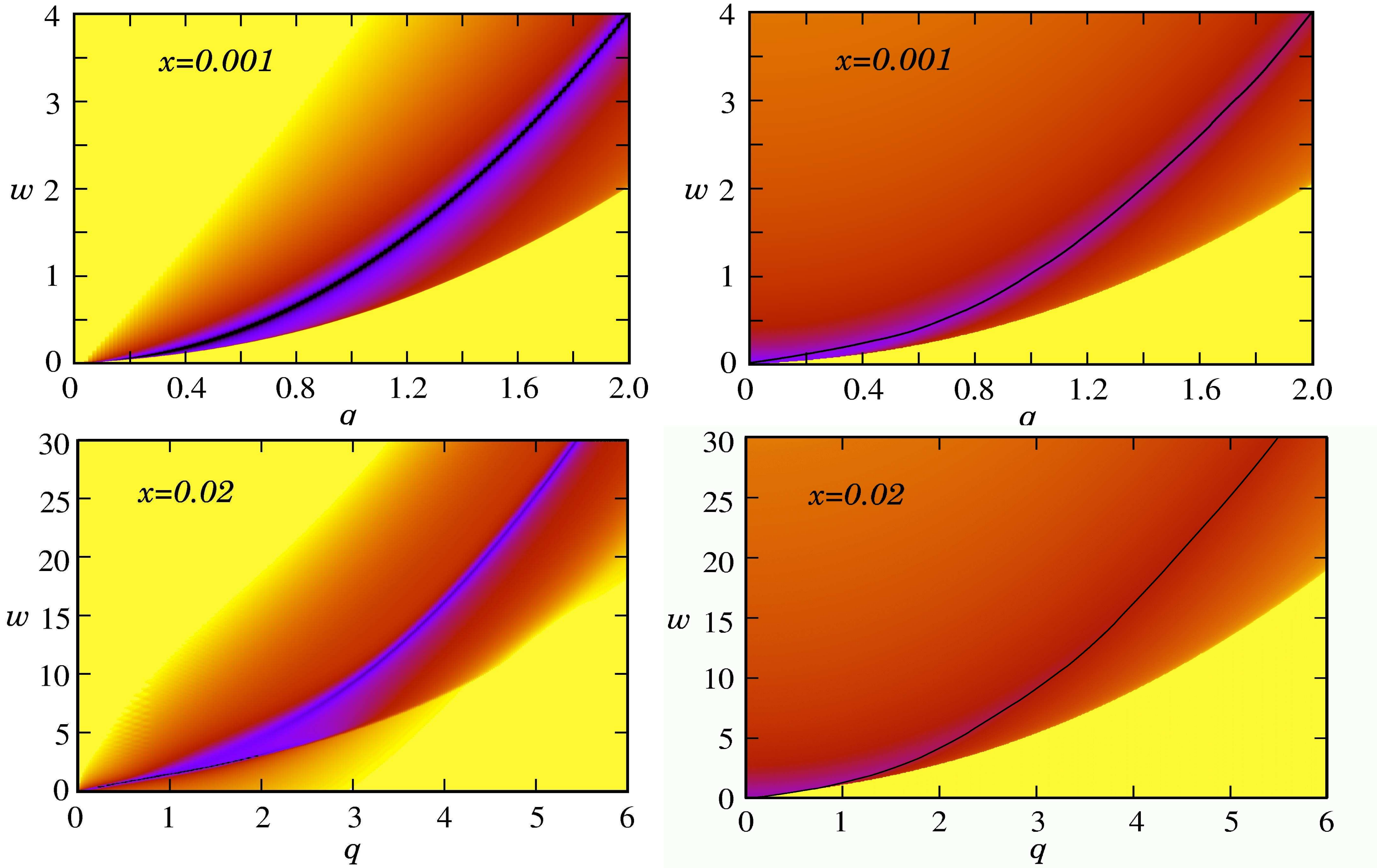
S(*k*) for Hard Spheres at several values of the gas parameter $x=10^{-4}$, 10^{-3}, 10^{-2} and 10^{-1}

S(*k*) for Soft Spheres at two values of the gas parameter $x=10^{-4}$ and 10^{-3}, for R=10a (SS10) and R=5a (SS5)
(upper and lower curves in each case)



Dynamic response function for hard spheres

CBF | Bogoliubov

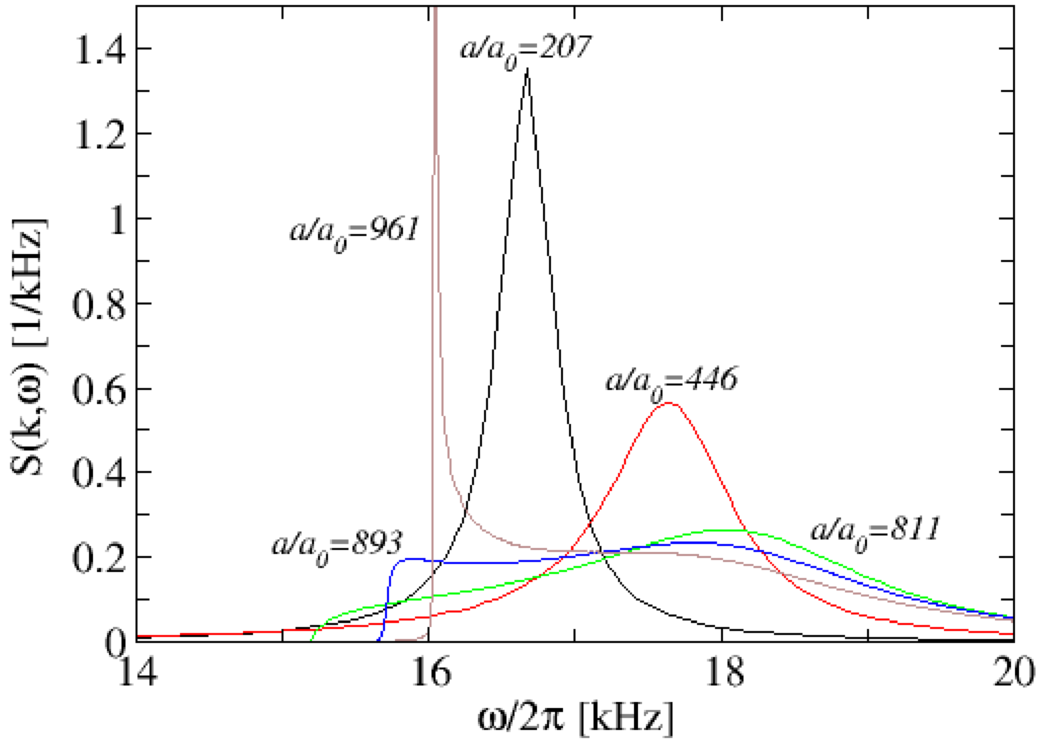


CBF results for Soft Spheres S(k,w) with R=3.5a

$$k = \frac{4\pi}{780} nm^{-1}$$

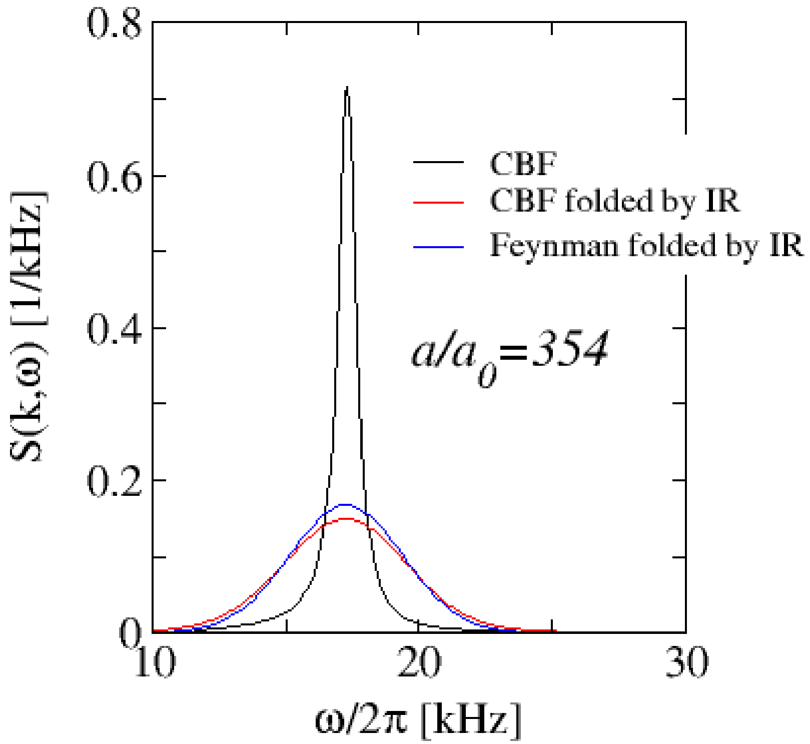
$$\rho = 7.6 \cdot 10^{13} cm^{-3}$$

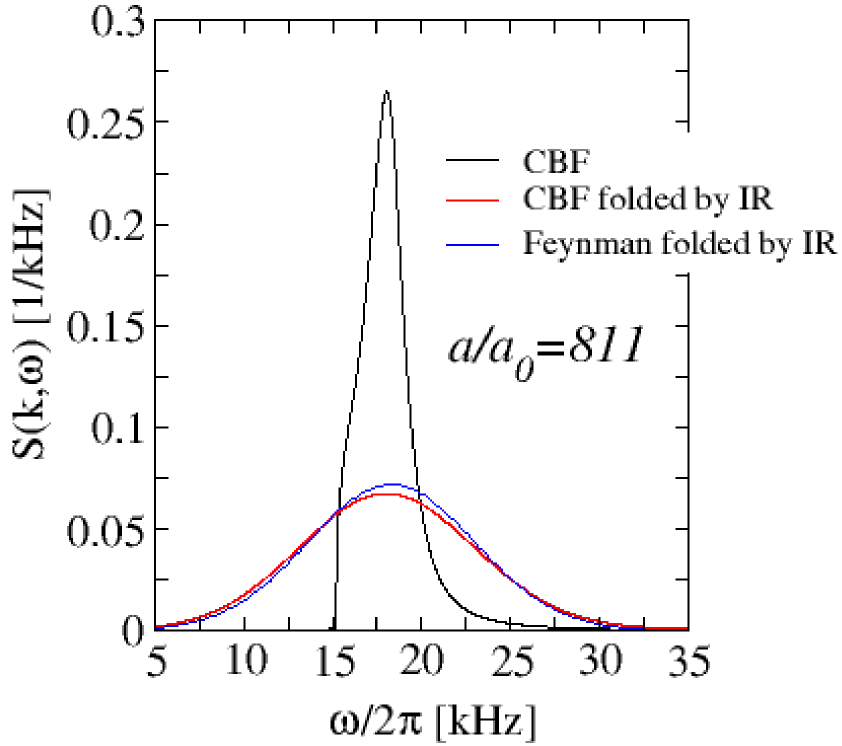
Soft Spheres with R=3.5a for different values of the scattering length



The position of the peak moves to higher energies with increasing scattering length, then disappears, and finally shifts to lower energies at even higher values of a

Impact of the Instrumental Resolution Effects for SS with R=3.5a

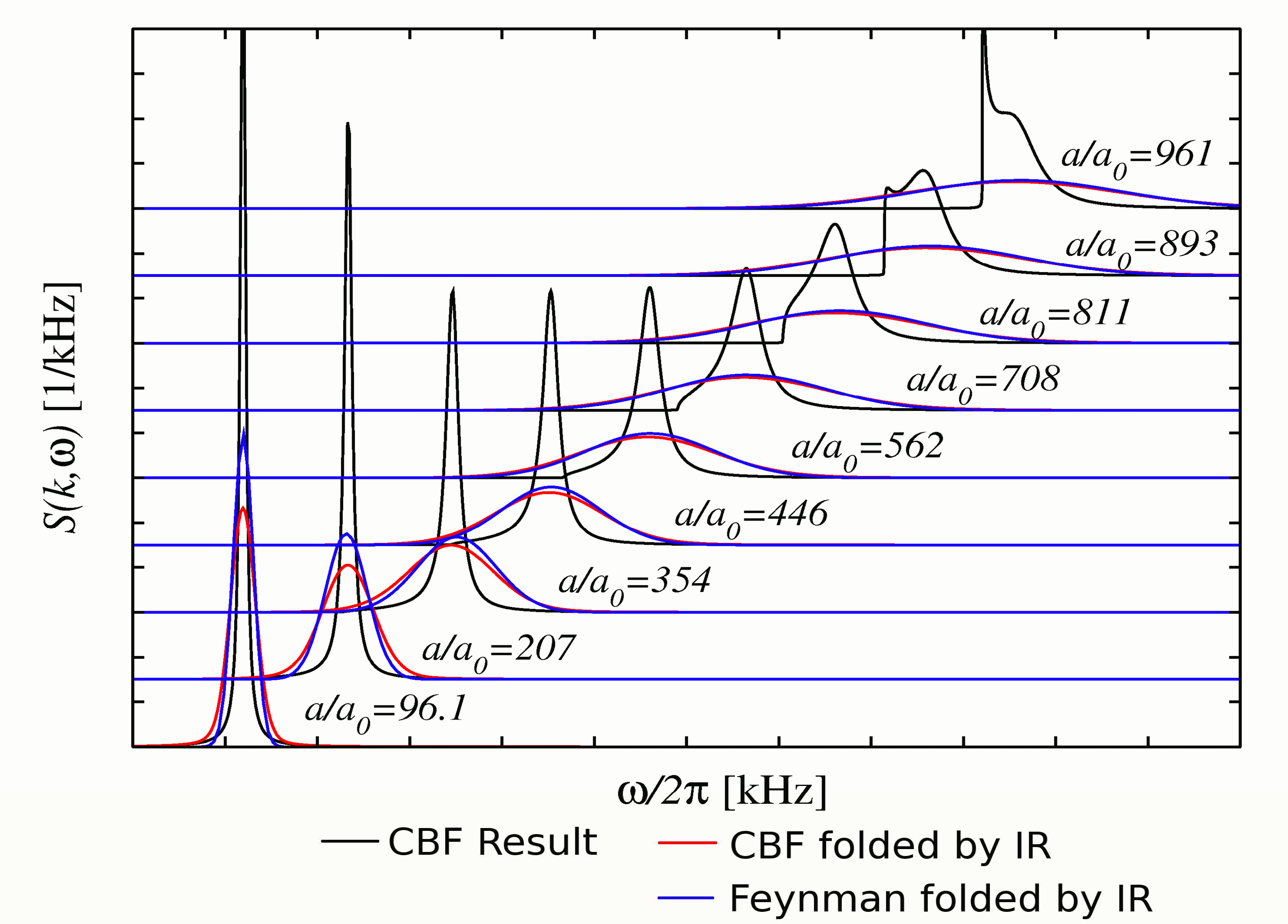




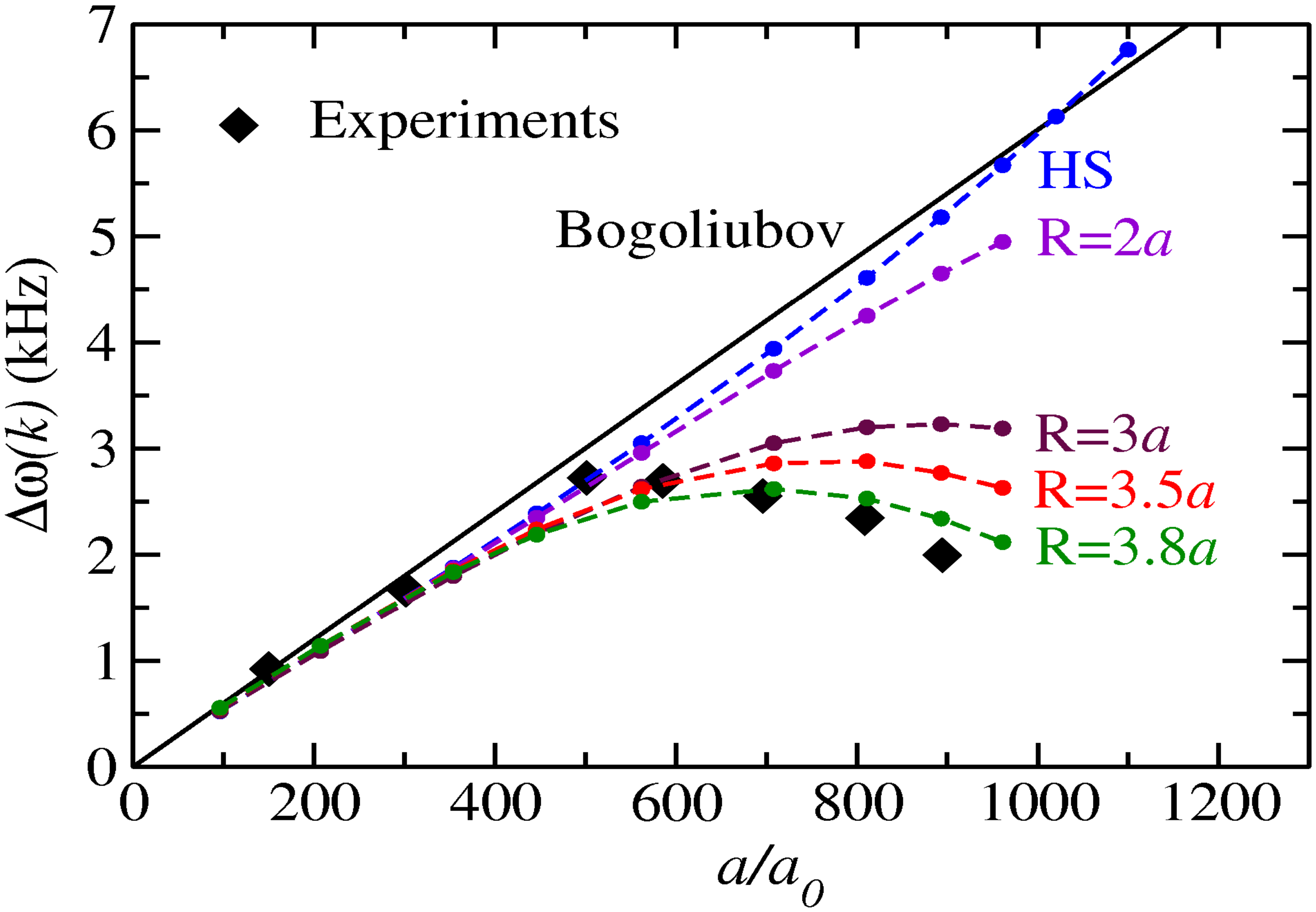
Instrumental resolution effects do kill all features in the response, leaving only a gaussian-like function that is almost identical to the Feynman approximation folded by the same function

Max. Differences between folded Feynman and folded CBF ~0.4 kHz

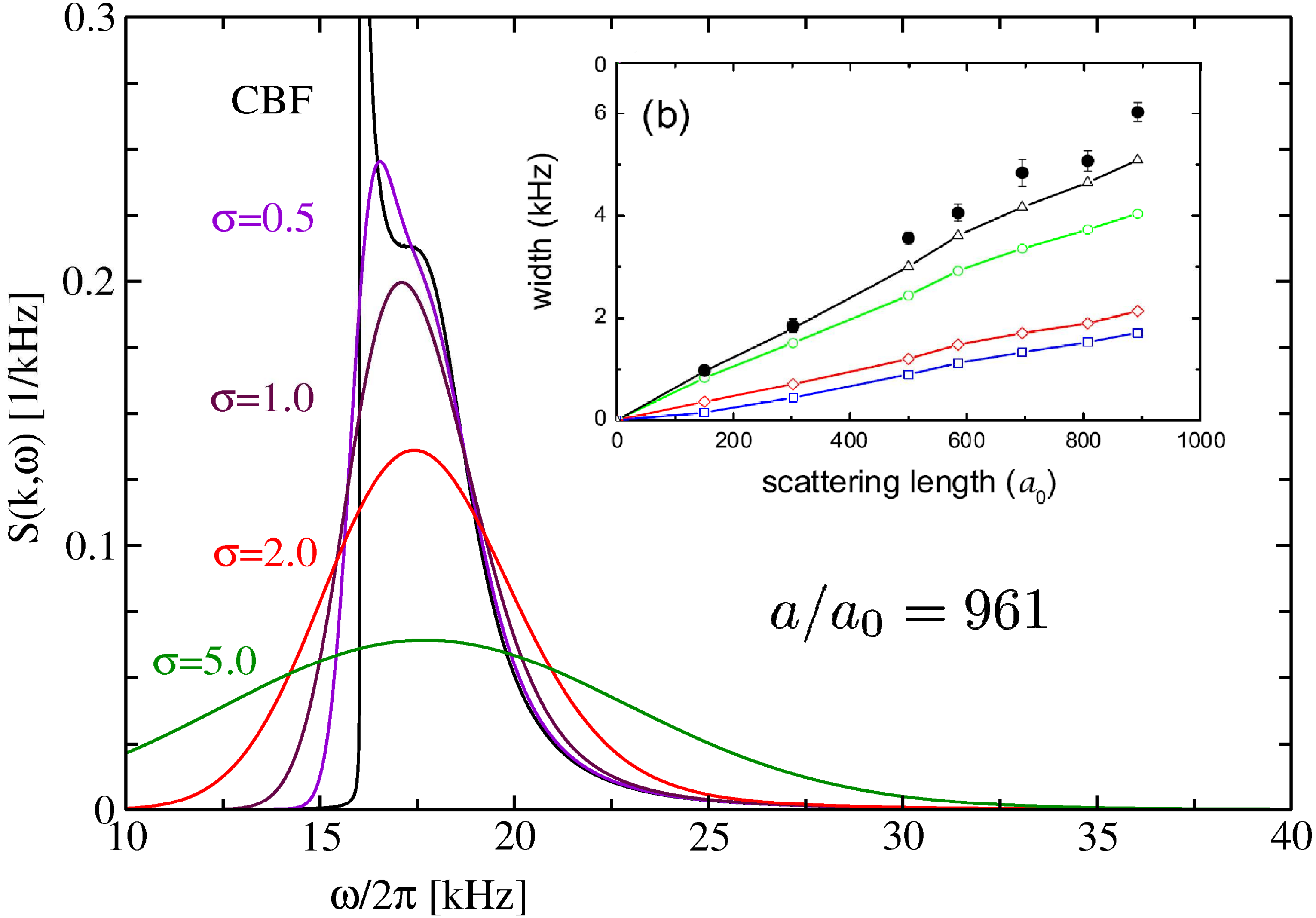
Impact of the Instrumental Resolution Effects for SS with R=3.5a



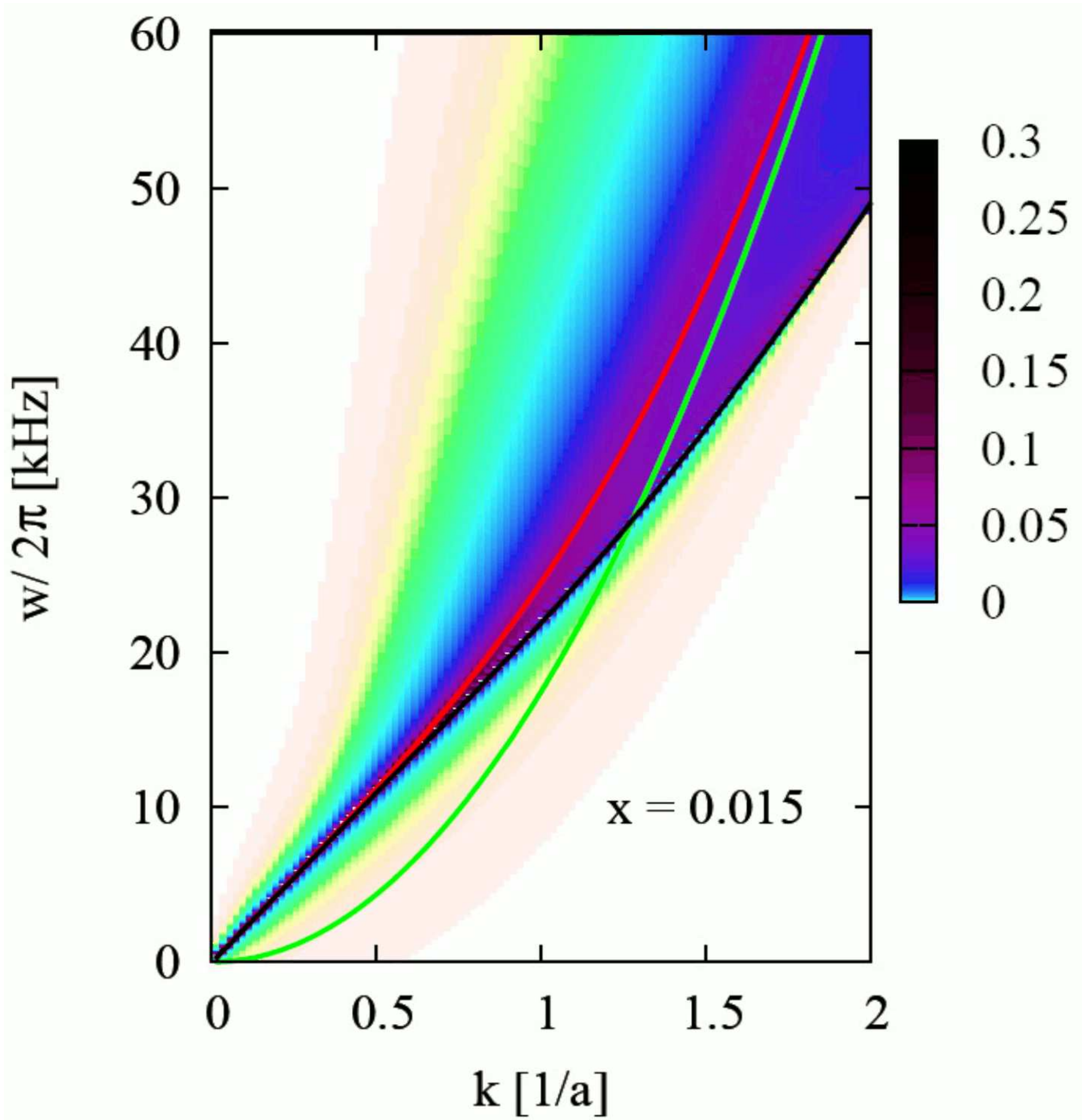
Results for Hard Spheres and Soft Spheres



Impact of the Instrumental Resolution Effects for SS with R=3.5a



S(k,ω) for Soft Spheres

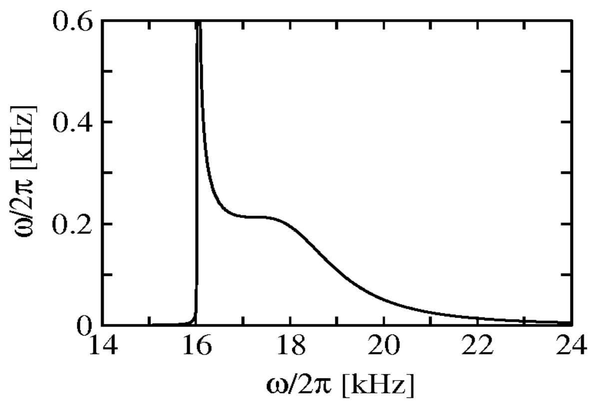


S(k,ω) map for a somewhat larger value of x.

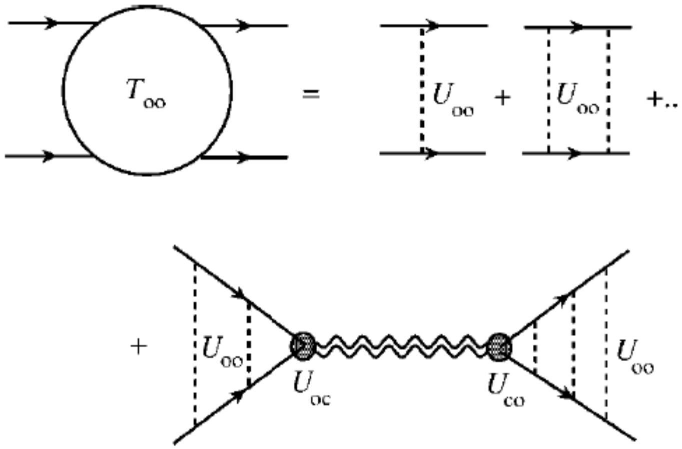
Green line: free

Red line: Feynman

Wider structures develop with increasing energy ω, which could be seen increasing the resolution



Effective interaction from the two-channel T-matrix approach



$$T_{oo} = \frac{\frac{4\pi}{m} a_{bg}}{\left(1 + \frac{\Delta\mu\Delta B}{q^2/m - \Delta\mu(B - B_0)}\right)^{-1} + i a_{bg} q},$$

$$T_{oo} = \frac{4\pi a}{m} \frac{1}{1 - \frac{1}{2} q^2 a r_{\text{eff}} + i q a},$$

$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right) \qquad r_{\text{eff}} = -\frac{2}{m \Delta\mu \Delta B a_{bg}}.$$

B_0= 155G, ΔB=11G , a_{bg}= -450a_0 and $\Delta\mu$/h=-3.26MHz/G

PHYSICAL REVIEW A **71**, 052713 (2005)

Multichannel scattering and Feshbach resonances: Effective theory, phenomenology, and many-body effects

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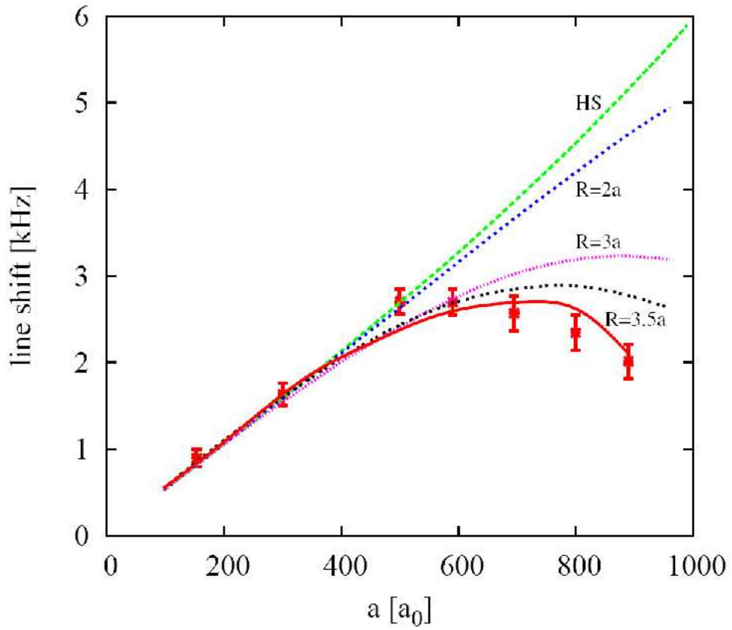
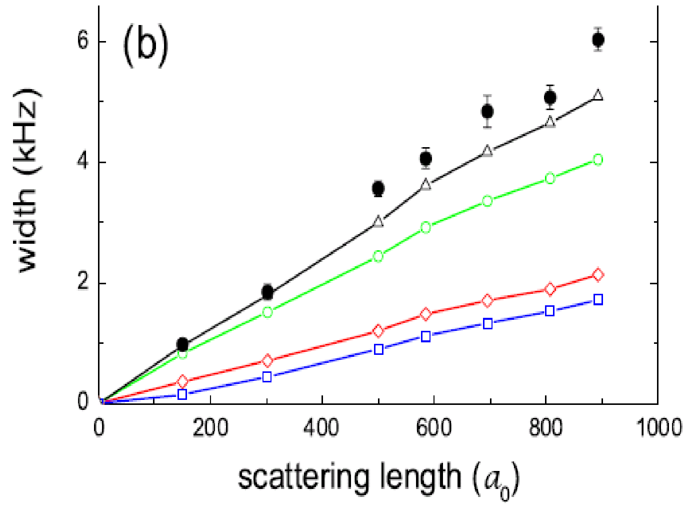
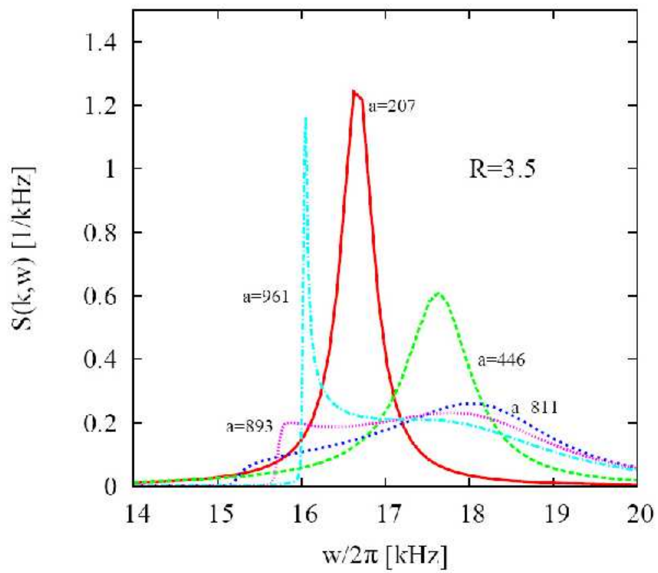
(Received 18 January 2005; published 26 May 2005)

Our results

From the effective range we get
the SC-potential range R=2.5a



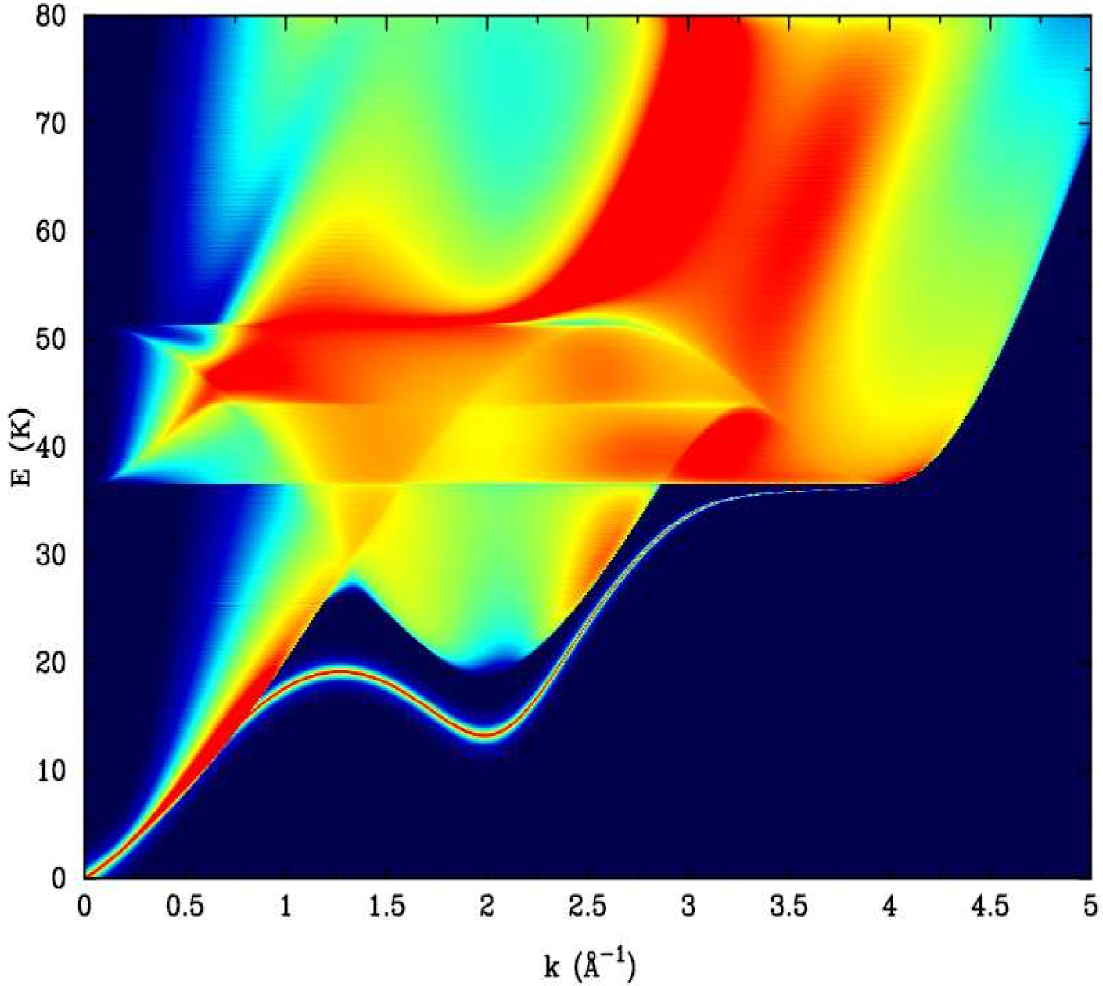




The half width increases from 0.5 kHz
to 3.5 kHz in the range 200<a<900 in
agreement with experiments.

Dynamic structure function in ^4He

Helium at zero pressure



Summary and Conclusions:

- We can use simple HNC/EL and CBF to describe quantum gases at the experimental values of x nowadays available, showing the departure from the universal regime.

- At least a two-parameter potential is required to get qualitative agreement with the experimental data for the excitation spectrum.

- Experiments have to improve considerably the resolution in order to see any detailed structure that goes beyond the Feynman approximation.

- Realistic potentials derived from the scattering theory beyond the scattering length approximation are needed to describe the system. *Work is being done along this line...*