Elementary Particle Physics: Assignment # 3
Due Monday Oct 24, 16:30 pm

1 An electron enters a region with a constant magnetic field perpendicularly to the magnetic field. Its trajectory is a circle and its spin precesses about the axis of the magnetic field. Find the frequency at which the electron circulates in its trajectory (this is the Larmor frequency). Find the frequency at which the spin precesses. Show that both frequencies are equal only if \( g = 2 \). Discuss what this means for the helicity of the particle. [Hint: The trajectory of the electron is described by the condition \( F = evB = m \frac{v^2}{R} \) where \( R \) is the radius of the trajectory, and \( v \) is the velocity of the electron. The precession of the spin is described by the equation \( \frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} \) where \( \vec{\mu} = -\frac{g}{2m} \vec{S} \) is the intrinsic magnetic moment of the electron.]

2 Probe the completeness relations (\( \xi_0 = -1 \) and \( \xi_1 = \xi_2 = \xi_3 = 1 \))

\[
\begin{align*}
2.1) \quad & \sum_{s=1,2} u^s(p)\bar{u}^s(p) = \not{\Phi} + m \\
2.2) \quad & \sum_{\lambda=0}^{3} \xi_{\lambda}(\epsilon^\ast_{\lambda}(q))^{\mu}(\epsilon_{\lambda}(q))^{\nu} = -g_{\mu\nu}
\end{align*}
\]

3 Show that the mass matrix \( M \) in a Majorana mass term

\[
-L_{\text{Maj}} = -\frac{1}{2} \bar{\nu}_{L,i}M_{ij}(\nu_{L,j})^{c} + h.c
\]

must be symmetric \( M_{ij} = M_{ji} \).