(1) Using that for $e^-(k)P(p) \rightarrow e^-(k')X$ scattering

$$|M^2| = \frac{e^4}{q^4} 4\pi M L_{\text{elec}}^{\mu\nu} W_{\mu\nu}^{\text{had}}$$

$$L_{\text{elec}}^{\mu\nu} = 2[k^\mu k'^\nu + k'^\mu k^\nu - (k.k')g^{\mu\nu}]$$

$$W_{\mu\nu}^{\text{had}} = W_1(\nu, q^2) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right] + \frac{W_2(\nu, q^2)}{M^2} \left[ p^\mu + \frac{(p.q)}{q^2} q^\mu \right] \left[ p^\nu + \frac{(p.q)}{q^2} q^\nu \right]$$

prove that in the Lab frame (initial proton at rest)

$$\frac{d\sigma}{dE'd\Omega'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{4}} \left[ W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2} \right]$$

(1)

where $E$ ($E'$) is the initial (final) electron energy, $d\Omega'$ is the solid angle for the final electron, $q = k - k'$, $\nu = \frac{E'}{M}$, and $M$ is the proton mass.

(2) Show that in $e^-(k)P(p) \rightarrow e^-(k')X$ scattering

(a) The invariant mass of the $X$ system, $M_X^2 = M^2 + 2M\nu + q^2$

(b) $x = \frac{-q^2}{2(p.q)}$ is restricted to the range $0 \leq x \leq 1$ if $M_X \geq M$.

(c) $y = \frac{(p.q)}{(p.k)}$ is restricted to the range $0 \leq y \leq 1$ [Hint: Use Lorentz invariance and work the proof in the lab frame]

(3) Consider the strange quark contribution to $F_{2}^{np}$ and $F_{2}^{en}$

$$\frac{1}{x} F_{2}^{np}(x) = \left(\frac{2}{3}\right)^2 [u^p(x) + \overline{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \overline{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \overline{s}^p(x)]$$

$$\frac{1}{x} F_{2}^{en}(x) = \left(\frac{2}{3}\right)^2 [u^n(x) + \overline{u}^n(x)] + \left(\frac{1}{3}\right)^2 [d^n(x) + \overline{d}^n(x)] + \left(\frac{1}{3}\right)^2 [s^n(x) + \overline{s}^n(x)]$$

Show that if $u^p(x) = d^p(x) \equiv u(x), d^p(x) = u^n(x) \equiv d(x)$ and $s^n(x) = s^n(x) \equiv s(x), \overline{u}^p(x) = \overline{d}^p(x) \equiv \overline{u}(x), \overline{d}^n(x) = \overline{u}^n(x) \equiv \overline{d}(x)$ and $\overline{s}^p(x) = \overline{s}^n(x) \equiv \overline{s}(x)$, the experimental results

$$\int_{0}^{1} dx F_{2}^{np}(x) = 0.18 \quad \text{and} \quad \int_{0}^{1} dx F_{2}^{en}(x) = 0.12$$

still implies that there is some momentum of the proton not carried by the quarks.