Elementary Particle Physics: Assignment # 1

Due MONDAY Oct 9th 4:10 pm (before starting class)

(1) For the following reactions write the quantum numbers Q,L,B,L_α ($\alpha = e, \mu, \tau$) , S (strangeness), C (charm) for all the states and use the conservation of quantum numbers described in class to discuss which processes are forbidden and which are allowed by some of the three types of interactions: strong, electromagnetic or weak and why. For processes allowed by weak interactions only, discuss which ones would be forbidden if neutrinos were massless, and which ones we do not know yet if they are allowed or not.

(a) $\mu^- \to e^- e^+ e^-$ (b) $n \to \mu^+ \pi^-$ (c) $n\, n \to p\, p\, e^-\, e^-$
(d) $n\, p \to p\, n\, \pi^+$ (e) $e^+ e^- \to \nu_\tau\, \bar{\nu}_\tau$ (f) $\Lambda_c^+ \to \Sigma^+ \pi^0$

(2) The momentum expansion of a free scalar complex field

$$\Phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2Ep}} \left[a_p e^{-ipx} + b_p^{s\,\dagger} e^{ipx}\right] \text{ with } E_p = \sqrt{|\vec{p}|^2 + m^2}$$

a_p and b_p verify: $\left[a_p, a_q^\dagger\right] = \left[b_p, b_q^\dagger\right] = (2\pi)^3 \delta^3(p-q)$ and all other commutators vanish.

The propagator is defined

$$i\Delta_F(x-y) \equiv \langle 0| T\left[\Phi(x)\Phi^\star(y)\right]|0\rangle \equiv \Theta(x_0-y_0)\langle 0|\Phi(x)\,\Phi^\star(y)|0\rangle + \Theta(y_0-x_0)\langle 0|\Phi(y)^\star\,\Phi(x)|0\rangle$$

Using the expansion of the fields and the commutation relations above show that

$$\Delta_F(x-y) = \Theta(x_0-y_0)\Delta_+(x-y) - \Theta(y_0-x_0)\Delta_-(x-y)$$

with

$$\Delta_\pm(x-y) = \mp i \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{\mp ip(x-y)}$$

where Δ_+ comes from the term $\langle 0|a_p a_p^\dagger|0\rangle$ and Δ_- comes from the term $\langle 0|b_p b_p^\dagger|0\rangle$

(3) In the chiral representation the 4-spinors for a fermion with momentum $\vec{p} = |\vec{p}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ with positive and negative helicity are:

$$u^{1,2}(p) = u^\pm(p) = \begin{pmatrix} \sqrt{E \mp |\vec{p}|}\,\xi_p^\pm \\ \sqrt{E \pm |\vec{p}|}\,\xi_p^\pm \end{pmatrix} \quad \text{with} \quad \xi_p^+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} \quad \xi_p^- = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

and the corresponding 4-spinors for the anti-fermion are: $v^{1,2}(p) = v^\pm(p) = \pm \begin{pmatrix} \sqrt{E \pm |\vec{p}|}\,\xi_p^\mp \\ -\sqrt{E \mp |\vec{p}|}\,\xi_p^\mp \end{pmatrix}$

<u>Using these expresions</u> evaluate <u>by direct calculation</u>

(4.1) $\bar{u}^s(p)u^r(-p)$ (4.2) $\bar{v}^s(p)u^r(-p)$

For the 4 possible combinations of $s, r = 1, 2$