Particle Physics: Assignment # 2

Due Monday 16/10/17, 4:10 pm

1. The transformation properties of the fermion and vector fields given in class were:

$$\begin{array}{ll} \mathcal{P}\,\psi(x)\,\mathcal{P}^{-1} = \alpha_P^{\psi}\,P\,\psi(x_P) & \mathcal{P}\,\bar{\psi}(x)\,\mathcal{P}^{-1} = \alpha_P^{\psi^*}\,\bar{\psi}(x_P)\,P \\ \mathcal{C}\,\psi(x)\,\mathcal{C}^{-1} = \alpha_C^{\psi}\,C\,\bar{\psi}^T(x) & \mathcal{C}\,\bar{\psi}(x)\,\mathcal{C}^{-1} = -\alpha_C^{\psi^*}\,\psi^T(x)C^{-1} \\ \mathcal{P}\,V^{\mu}(x)\,\mathcal{P}^{-1} = P^{\mu}_{\nu}V^{\nu}(x_P) & \mathcal{C}\,V^{\mu}(x)\,\mathcal{C}^{-1} = -(V^{\nu}(x))^* \end{array}$$

with $P^{\nu}_{\nu} = \mathrm{diag}(1,-1,-1,-1)$, $P = \gamma^0$ and $C = -i\gamma^2\gamma^0$.

Using them derive the transformation properties under Parity ($\mathcal{P}$), Charge Conjugation ($\mathcal{C}$) and the product of both ($\mathcal{CP}$) of the following four bilinears (a and b are two type of fermions) (you can use that the γ^{μ} transform as a vector under parity).

$$\begin{array}{ll} 1)\ \bar{\psi}_a(x)\psi_b(x) & 2)\ \bar{\psi}_a(x)\gamma_5\psi_b(x) \\ 3)\ \bar{\psi}_a(x)\gamma^{\nu}\psi_b(x) & 4)\ \bar{\psi}_a(x)\gamma^{\nu}\gamma^5\psi_b(x) \end{array}$$

With the resuts above check whether the following Lagrangiangs are invariant under C, P and CP

$$\begin{array}{l} \mathcal{L} = -B\ \bar{\psi}_a(x)\gamma^{\mu}\psi_a(x)A_{\mu} \\ \mathcal{L} = -B\ \bar{\psi}_a(x)\gamma^{\mu}(1-\gamma^5)\psi_a(x)A_{\mu} \\ \mathcal{L} = -D\ \bar{\psi}_a(x)\gamma^{\mu}\psi_b(x)V_{\mu} + h.c. \\ \mathcal{L} = -D\ \bar{\psi}_a(x)\gamma^{\mu}(1-\gamma^5)\psi_b(x)V_{\mu} + h.c. \end{array}$$

A^{μ} is a real vector field, while V^{μ} is a complex vector field. B is real (as required by reality of the Lagrangian). D can be complex.

2. The interaction Lagrangian for a neutral scalar particle H with field ϕ with two types of fermions, e with field ψ and μ with field χ, reads

$$\mathcal{L}_I = -\lambda_e\phi(x)\bar{\psi}(x)\psi(x) - \lambda_{\mu}\phi(x)\bar{\chi}(x)\chi(x)$$

Using Wick's theorem and the expansion of the fields in terms of creation an annihilation operators explicitly derive the Feynmann amplitude (at the lowest non-vanishing order) for the process

$$e^+(p1,s1)\,e^-(p2,s2) \to \mu^+(q1,r1)\,\mu^-(q2,r2)$$

Hint: Notice that the vertices scalar-fermion-fermion in the Lagrangian do not mix electrons and muons so there is only one contribution.