Elementary Particle Physics: Assignment # 5
Due Monday Nov 13th 4:10 pm

(1) Quarks are in the fundamental representation of color SU(3) (triplet) and antiquarks are anti-triplets. A baryon is a color singlet formed by three quarks while a meson is a color singlet formed with a quark and an antiquark. A pentaquark is a physical state (ie a color singlet) formed with 4 quarks and one antiquark. By explicit composition of the SU(3) representation of the five components, show that a pentaquark could be a "molecule" of a baryon and a meson, or a purely strong bound state of the five components (ie not subcomposed of colour singlet states).

(2) Under an $SU(3)_C$ gauge transformation of parmeters $\theta_a(x)$ $(a = 1, \ldots 8)$ the field of the quark q of color i transforms as the triplet (fundamental) representation

$$\psi'_{q_i}(x) = \left(\exp[i\frac{\lambda^a}{2}\theta_a(x)]\right)_{ij} \psi_{q_j}(x)$$

Show that the QCD lagrangian

$$\mathcal{L}^{QCD} = \bar{\psi}_{q_i}(x)\left(i\gamma^\mu D_{\mu ij} - m_q\delta_{ij}\right)\psi_{q_j}(x) - \frac{1}{4}G^{\mu\nu}_a(x)G_{\mu\nu,a}(x)$$

with

$$D^\mu_{ij}\psi_{q_j}(x) = \left(\delta_{ij}\partial^\mu + ig_s\frac{\lambda^a_{ij}}{2}G^\mu_a(x)\right)\psi_{q_j}(x)$$

and

$$G^{\mu\nu}_a(x) = \partial^\mu G^\nu_a(x) - \partial^\nu G^\mu_a(x) - g_s f_{abc}G^\mu_a(x)G^\nu_b(x)$$

is gauge invariant under $SU(3)_c$ if the gluon field transform as the adjoin representation of $SU(3)_c$

$$G^\mu_a(x)' = G^\mu_a(x) - \frac{1}{g_s}\partial^\mu\theta_a(x) - f_{abc}\theta_b(x)G^\mu_c(x)$$