1. Using the parton model, find the structure functions F_1, F_2, F_3 for the processes:

$$\begin{aligned}\nu p &\to e^- + X \\ \nu n &\to e^- + X \\ \bar{\nu} p &\to e^+ + X \\ \bar{\nu} n &\to e^+ + X\end{aligned}$$

in terms of the Parton Distribution Functions ($u(x)$, $\bar{u}(x)$, etc.). Consider only u, d, s and c quarks, and assume $\cos\theta_c = 1$.

Prove the Adler sum rule

$$\int \frac{\mathrm{d}x}{x}(F_2^{\bar{\nu}p}(x) - F_2^{\nu p}(x)) = 2$$

and the Gross-Llewellyn-Smith sum rule

$$\int_0^1 \mathrm{d}x[F_3^{\nu p}(x) + F_3^{\nu n}(x)) = -6$$

2. Using the Feynman Rules for QCD, determine the imaginary part of the self-energy $\Sigma(\not{p})$ of quark of mass m

$$\Sigma(\not{p}) = \not{p}\Sigma_1(p^2) + \Sigma_2(p^2)$$

Use the dispersion relations to reconstruct the full Σ_1 and Σ_2 up to constants.

3. Prove that, in the limit in which the momentum transfer is much smaller than M_W, the weak interaction is reduced to the Fermi Lagrangian, and find the value of the Fermi Constant as a function of: the fine structure constant $\alpha = e^2/(4\pi)$, the W-boson mass, M_W, and the Weinberg angle $\sin^2\theta_W$. Given $G_F = 1.16637 \times 10^{-5} GeV^{-2}$, $\alpha \simeq 1/137$, $\sin^2\theta_W \simeq 0.2259$, find an approximate value for M_W.