Exercices and questions for lectures 3 & 4

1.- Find the energy of the state $(a_k^\dagger)^2|0\rangle$.

2.- Find in terms of creation and annihilation operators the expression for the the normal ordered momentum operator $:\mathcal{P}^i:$.

3.- Find the normalization constant N for the spinors such that $\bar{u}u = -\bar{v}v = 2m$.

4.- Verify the relations (128).

§ **5.-** Verify that the helicity operator $h(\vec{p})$ commutes with the Dirac hamiltonian $H = \vec{\alpha}\vec{p} + \beta m$. Prove by direct diagonalization that the eigenvalues of h are ± 1.

6.- Prove that $u(-\vec{p}, -\lambda) = i\gamma^0 u(\vec{p}, \lambda)$ and $v(-\vec{p}, -\lambda) = i\gamma^0 u(\vec{p}, \lambda)$, where λ is the helicity.

7.- Write the lagrangian for a complex scalar field of mass m coupled to electromagnetism via minimal coupling (covariant derivative). Verify that the field and its conjugate obey equations of motion with opposite signs for the coupling to the external field.

8.- Write the expansion in creation and annihilation operators for a free complex scalar field. Can you find a conserved Noether current associated to phase invariance? Interpret the correponding conserved charge if it exists.

9.- Find the solution for the spinors $u(\vec{p}, \lambda)$ corresponding to a Majorana fermion in the Majorana representation.

§ **10.-** Derive the commutation relations (181). Show, by calculating the norm of the corresponding state, that time-like polarization states are of negative norm.

§ **11.-** Derive the Hamiltonian (186).

12.- Verifiy the equation of motion of electromagnetism in the presence of matter fields (198). Verify that they correspond to the usual Maxwell equations (written in the gaussian system of units) when expressed in terms of the electric and magnetic fields.

13.- Construct the angular momentum generators and verify that the photon field describes a particle of spin $j = 1$.