11 Strong interactions: the Yukawa theory and other effective models

11.1 Confinement

It is a well established fact that quarks and gluons are confined inside hadrons. The force between two quarks is so strong that they cannot be separated. Quarks and gluons do not exist as asymptotic states. Furthermore, the mass of the hadrons themselves (protons for instance) has nothing (or very little) to do with the masses of the quarks. Recall that a typical hadron has a mass of order 1 GeV, while light quarks have masses of the order of few GeV. Thus the hadron masses must be dynamically generated due to the effect of very large quantum corrections that generate a dynamical scale. Let us call this scale Λ_{QCD}. All hadron masses, at least for light quarks, are proportional to this dynamical scale. We shall see later that for heavy quarks the situation is slightly different.

Strong interactions have thus a well defined scale Λ_{QCD} that sets a natural scale in the theory. Well above Λ_{QCD}, that is at short distances, perturbation theory makes sense. Of course perturbative QCD at large enough energies (very short distances) describes a world of quasi-free quarks, interacting with Coulomb-like forces, not very different from the QED world. We know very well that hadronic physics is a very different world with quarks are confined into colorless hadrons. As soon as $q^2 \sim \Lambda_{QCD}^2$ perturbation theory is unreliable. It simply cannot explain confinement.

What does confinement actually mean? One popular interpretation is that 'it is not possible to detect an isolated quark or gluon'. The problem with this definition of confinement is that in electromagnetism, which is certainly "not confining", it is not possible to detect an isolated electron either. Electromagnetism (as well as Quantum Chromodynamics) is long-range (mediated by a massless particle) and plagued (just as QCD) with infrared divergences. Both QED and QCD observables have to be inclusive enough. There is one difference, of course, and that is that photons have no $U(1)$ charge, while gluons do carry $SU(3)$ charges and interact among themeselves. Far away from the interaction point you can hope to be able to measure the electric charge carried by one particle (or rather what the experimental resolution defines as a particle which is actually the electron surrounded by a cloud of soft photons), but even if you were able to construct a detector that measured color you probably would not be able to identify in any way the color of the quark itself.

In fact, there is another definition of confinement that tells to you that the chances of actually seeing a (gluon-dressed) quark are small: 'there is a force between quarks that does not decrease with distance'. There is indeed phenomenological evidence (which is supported by lattice analysis) that the interquark potential in QCD is of the form

$$V(r) \sim a\Lambda_{QCD}^2 r - \frac{b}{r} + \dots \tag{453}$$

The first term is a confining quark potential. The constant a has to be ~ 1 because Λ_{QCD}^2 is the only dimensional quantity at our disposal. The Coulombic part is called the Lüscher term and plays a crucial theoretical role even if it is not very important in heavy quark spectroscopy in practice.

We like this second definition of confinement better, because the first one is far too imprecise. In order to see that it is useful to recall a toy example suggested by Georgi. Before that we need to introduce the concept of "effective" or "running" coupling constant. Strong interaction corrections (at the fundamental level of quarks and gluons) have a constant similar to the fine structure constant α, called α_s. However, α_s is at least one order of magnitude larger that α. Furthermore, a good way of summing up the leading quantum corrections (that in QCD are very large as mentioned)is provided by defining a "running" coupling $\alpha_s(\mu)$ that actually depends on the typical energy of the process.

Imagine now a world in which we tune Λ_{QCD} in such a way that $\alpha_s(1\text{GeV}) = 1/137$. Since the coupling constant is so small, perturbation theory works wonderfully at such energies. The proton would be a bound state of quarks (bound by Coulomb-like forces that is) with mass roughly $3m_q$. Its size would be dictated by the Bohr radius, about 1000 times the size it has in our world. The inhabitants of this world would certainly not understand the first definition of confinement. The reason is that using

$$\alpha_s(\mu) = \frac{-\pi}{\frac{\beta_1}{2}\log\frac{\mu^2}{\Lambda^2}} \tag{454}$$

we obtain that the confinement radius would be $\sim \Lambda^{-1} \sim 10^{21}$cm. They would see 'free' quarks as you see electrons.

Even in our world the situation is somewhat similar to that of the toy world for very heavy quarks. Indeed $\alpha_s(m_t)$ is small (say ~ 0.1). The Bohr radius is $r_0 \sim 10^{-2}$ fm, much smaller than Λ_{QCD}^{-1}. The coulombic part of the interquark potential largely dominates. (At such short distances the linearly rising potential is not at work, the leading confinement effects are $\sim r^3$, as discussed by Leutwyler some time ago, but they can be safely neglected at first approximation.)

Bottom and charm are in a somewhat intermediate position. $\alpha_s(m_b)$ is still relatively small. The Bohr radius is 10^{-1} fm, smaller but comparable to Λ_{QCD}^{-1}. Spectroscopy is basically perturbative, at least for the lowest levels, but some non-perturbative effects are visible. Charm is really no-man's land. Both perturbative and non-perturbative effects compete even for the ground state $n = 1$. For light quarks the Bohr radius is several fm and the confining potential is fully at work.

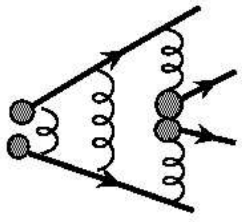

Figure 3: The QCD string.

The existence of a confining potential leads to very large multiplicities and jets. One can imagine a quark-antiquark being formed at a primary vertex then moving apart. Part of their kinetic energy is deposited in the interquark potential as they move away. Very quickly a separation r_m is reached where the energy deposited is enough to form a new quark-antiquark pair,

$$\Lambda_{QCD}^2 r_m \simeq 2m_q, \tag{455}$$

at that moment the quark-antiquark 'string' breaks and the process is repeated until the average relative momentum is small enough and hadronization takes place.

There is a lot of physics in the string picture. We can think of color forces being confined in some sort of tube or string joining the two moving quarks. The chromodynamic energy is thus stored in a relatively small region of space-time. If this picture is correct we should expect hadronization to take place in this region in preference to any other. This is indeed the case; in three jet events (which originate from $\bar{q}qg$, with a hard gluon) there is a clear enhancement of soft gluon and hadron production in the regions between color lines (representing the gluon by a double color line, or $\bar{q}q$ state), and a relative depletion in other regions. This phenomenon is called color coherence.

11.2 Modelling nucleon interactions: the sigma model

If quarks and gluons are confined inside hadrons (mesons and baryons) and gluons are the carriers of strong interactions, how do nucleons interact? How can we describe the familiar nuclear forces in terms of quarks and gluons?

Due to confinement the only possibility of interaction is via the exchange of mesons. In this way, two quarks are never 'too far away' and one gluon never need to 'fly' far away. Thus we want to construct the simplest possible model that describes meson exchange between nucleons and is compatible with the symmetry principles of QCD. It will nevertheless still be a toy model (realistic nuclear models are very complicated).

Isospin is an obvious symmetry of strong interactions, so let us write the nucleon doublet

$$\psi = \begin{pmatrix} \psi_p \\ n \end{pmatrix} \tag{456}$$

The kinetic term (neglecting the nucleon mass, for a second) will just be

$$L = i\bar{\psi}\ \partial\!\!\!/\psi = i\bar{\psi}_L\ \partial\!\!\!/\psi_L + i\bar{\psi}_R\ \partial\!\!\!/\psi_R. \tag{457}$$

This term is invariant under separate $SU(2)$ global (i.e. x-independent) transformations for left and right fields

$$\psi(x)_L \to \Omega_L\psi(x)_L, \qquad \psi(x)_R \to \Omega_R\psi(x)_R, \tag{458}$$

namely it has a global symmetry group $SU(2)_L \times SU(2)_R$. Including a common mass for the nucleon

$$L = i\bar{\psi}_L\ \partial\!\!\!/\psi_L + i\bar{\psi}_R\ \partial\!\!\!/\psi_R - m_N\bar{\psi}_L\psi_R - m_N\bar{\psi}_R\psi_L, \tag{459}$$

breaks this group to these transformations where $\Omega_L = \Omega_R$, this is the 'diagonal' or 'vector' subgroup $SU(2)V$ of $SU(2)_L \times SU(2)_R$, and is the familiar isospin. However, as said, QCD has the larger $SU(2)_L \times SU(2)_R$ group.

Instead of introducing a mass by hand, let us consider

$$L = i\bar{\psi}_L\ \partial\!\!\!/\psi_L + i\bar{\psi}_R\ \partial\!\!\!/\psi_R - g\bar{\psi}_L\Sigma\psi_R - g\bar{\psi}_R\Sigma^\dagger\psi_L. \tag{460}$$

if $\Sigma = m_N I$ this reduces to the previous lagrangian. However, the full $SU(2)_L \times SU(2)_R$ global symmetry can be preserved if Σ transforms under this group as

$$\Sigma \to \Omega_L\Sigma\Omega_R^\dagger. \tag{461}$$

Infinitessimally these transformations read

$$\delta\psi_L = i\epsilon_L^a T^a \psi_L, \quad \delta\psi_R = i\epsilon_R^a T^a \psi_R, \quad \delta\Sigma = i\epsilon_L^a T^a \Sigma - i\epsilon_R^a \Sigma T^a, \tag{462}$$

with $T^a = \tau^a/2$ (τ^a are the Pauli matrices).

Now we shall restrict Σ (8 independent parameters in principle) to be of the form

$$\Sigma(x) = \sigma(x) I + i\pi^a \tau^a. \tag{463}$$

It is not difficult to see that actually,

$$\Sigma = \sqrt{\sigma^2 + \vec{\pi}^2} U, \tag{464}$$

with U unitary, so this form is actually preserved by the unitary transformations in $SU(2)_L \times SU(2)_R$.

In term of the σ and π fields, the previous lagrangian can now be written as

$$L = i\bar{\psi} \not{\partial}\psi - g\sigma\bar{\psi}\psi + ig\vec{\pi}\bar{\psi}\vec{\tau}\gamma_5\psi. \tag{465}$$

This lagrangian was first written by Gell-Mann and Levy. We can couple external fields, such as electromagnetism, or weak interactions by just replacing the normal derivative by the covariant derivative. For isntance, for electromagnetism

$$\partial_\mu \to D_\mu = \partial_\mu + ieQA_\mu. \tag{466}$$

For weak interactions

$$\partial_\mu \to D_\mu = \partial_\mu + igP_L \frac{\vec{\tau}}{2} \vec{W}_\mu, \tag{467}$$

where P_L is the left projector.

The previous lagrangian can be complemented with the usual kinetic terms for the σ and $\vec{\pi}$ fields and a potential term. Both must be invariant under the global group. It is not difficult to see that this condition is atomatically fulfilled by

$$L_\Sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - V(\sigma^2 + \vec{\pi}^2). \tag{468}$$

It is clear that in order to give nucleons a mass we need to generate a vacuum expectation value for the σ field. Let us take the potential to be

$$V(\sigma^2 + \vec{\pi}^2) = \frac{\lambda}{4}[(\sigma^2 + \vec{\pi}^2) - F_\pi^2]^2. \tag{469}$$

Then the minimum is attained by $\sigma^2 + \vec{\pi}^2) = F_\pi^2$. We can use the symmetry now to select

$$v = \langle 0|\sigma|0\rangle = F_\pi, \qquad \vec{\pi} = 0. \tag{470}$$

All solutions are however physically strictly equivalent.

We shift the sigma field in order to have the usual expansion in creation and annihilation operators

$$\sigma \to F_\pi + \sigma. \tag{471}$$

Written in terms of the shifted field, the lagrangian reads

$$L = i\bar{\psi}\ \partial\!\!\!/\psi - gF_\pi\bar{\psi}\psi - g\sigma\bar{\psi}\psi + ig\vec{\pi}\bar{\psi}\vec{\tau}\gamma_5\psi \tag{472}$$

$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 + 2F_\pi\sigma)^2. \tag{473}$$

We are now in a position to "read" the physical content of the theory. For quantum number reasons it is natural to identify the π with the triplet of pions, the σ would then be the lightest meson with 0^+ quantum numbers. The nucleon mass is $m_N = gF_\pi$, the pion-nucleon coupling is thus $g_{\pi NN} = m_N/F_\pi$. In this model, pions are massless in this model, while the σ mass is $m_\sigma^2 = 2\lambda F_\pi$.

Can we correct the fact that pions are massless here while they are not in the real world? Certainly, but not keeping the whole $SU(2)\times SU(2)$ invariance. We can for instance add by hand a term that breaks this symmetry, but respecting $SU(2)_V$ (which one?). Then pions are massive and this corresponds to the familiar Yukawa potential of nuclear physics due to one-pion exchange

$$V(r) \sim \frac{1}{r}e^{-m_\pi r}. \tag{474}$$

Finally we note that the Noether current associated to the transformations (see problem 6)

$$\delta\psi = i\gamma_5\epsilon_5^a T^a\psi \tag{475}$$

contains a piece of the form

$$j_{5a}^\mu = -F_\pi\partial^\mu\pi_a + ... \tag{476}$$

Then

$$\langle 0|j_{5a}^\mu(0)|\pi_b(p)\rangle = iF_\pi p^\mu\delta_{ab}. \tag{477}$$