3 Gauge structure of the Standard Model

There are six known leptons $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$. Of those not all helicity states are found in nature (barring the possibility of a small mass for the neutrinos). Only left-handed neutrinos or right-handed antineutrinos seem to exist. Right-handed neutrinos (as required if they have a mass, however small, and they are not Majorana particles) would be 'sterile' anyway, i.e. not coupling to the Standard Model gauge fields.

These particles interact through electromagnetism and weak interactions, or only weakly (neutrinos). From the previous discussions we know that we can group the left handed fields in doublets

$$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad etc. \tag{28}$$

and singlets

$$e_R, \qquad etc. \tag{29}$$

If we want to describe electromagnetism in an unified way with weak interactions, we need to add a new gauge group $U(1)$. Using the third component of the $SU(2)_L$ group will obviously not work beacuse electromagnetism is not chiral. The new $U(1)$ must therefore act on left and right components. However, it would be premature to identify this new group with $U(1)_{em}$ since for the left part there is certainly the possibity of mixing with the neutral component of $SU(2)$. We shall call this new group $U(1)_Y$ where Y is the corresponding generator that we call hypercharge and of course we have to distinguish between Y_L and Y_R. We shall denote the corresponding gauge field by B_μ.

The basic building block will thus be the covariant derivative

$$D_\mu = \partial_\mu + ig\vec{W}_\mu \vec{T} + ig' B_\mu Y \tag{30}$$

where $T^a = \tau^a/2$ and $[T^a, T^b] = i\epsilon_{abc} T^c$.

If we want this theory to contain electromagnetism, it must be the case that, acting on doublets

$$eA_\mu Q \subset gW_\mu^3 T^3 + g' B_\mu Y_L \tag{31}$$

and on singlets

$$eA_\mu Q \subset g' B_\mu Y_R \tag{32}$$

This immediately shows that $Y_R \propto Q$. As for Y_L we note that the only linearly independent generator is proportional to the identity I and, in fact we can take $Y_L = Q - T^3$.

We allow for some mixing of the neutral fields. The corresponding mixing must be described by an orthogonal matrix so as to preserve the form of the kinetic terms (the usual ones for a gauge theory). We introduce a mixing angle θ_W (the so-called Weinberg angle) and denote $c_W = \cos\theta_W$, $s_W = \sin\theta_W$. Then, the photon will be given by the combination

$$A_\mu = s_W W^3_\mu + c_W B^\mu \tag{33}$$

The orthogonal combination is denoted by Z_μ

$$Z_\mu = c_W W^3_\mu - s_W B^\mu \tag{34}$$

The identification is the correct one provided that

$$g = \frac{e}{s_W} \qquad g' = \frac{e}{c_W} \tag{35}$$

We give the inverse relation

$$W^3_\mu = s_W A_\mu + c_W Z_\mu \tag{36}$$

$$B_\mu = c_W A_\mu - s_W Z_\mu \tag{37}$$

Let us check that for the couplings of the left handed fermions. Making all replacements, the l.h.s. of (31) reads

$$(g s_W A_\mu + g c_W Z_\mu) \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + (g' c_W A_\mu - g' s_W Z_\mu) \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}, \tag{38}$$

this leads to

$$\begin{pmatrix} -\frac{1}{2}\frac{e}{s_W c_W} Z_\mu & 0 \\ 0 & -eA_\mu - \frac{1}{2} e \frac{1-2s_W^2}{s_W c_W} Z_\mu \end{pmatrix}. \tag{39}$$

Let us now write the Z couplings to right handed fermions

$$g' B_\mu Y_R = (eA_\mu - e\frac{s_W}{c_W} Z_\mu) Y_R \tag{40}$$

We see that indeed $Y_R = Q$. We also see from the previous expression and the one corresponding to the left couplings that the Z_μ couplings are given by

$$\frac{e}{s_W c_W}(T^3 - s_W^2 Q) \tag{41}$$

The charged currents are much simpler. They only involve couplings between left-handed currents. The corresponding couplings have been already given in the previous sections.

In order to continue with our exploration of the weak interactions, lets now discuss the couplings to quarks. These also come in three families that we separate into left-handed doublets and right-handed singlets

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L \tag{42}$$

$$(u_R, d_R) \qquad (c_R, s_R) \qquad (t_R, b_R) \tag{43}$$

The corresponding charge matrix is

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \tag{44}$$

so

$$Y_R = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & -\frac{1}{6} \end{pmatrix} \tag{45}$$

and $Y_R = 2/3$ for u-type quarks and $Y_R = -1/3$ for d-type quarks.

To get some practice we can derive the Z_μ coupling to d-type quarks

$$\frac{e}{6s_W c_W}(-3 + 2s_W^2)\bar{d}_L \gamma^\mu Z_\mu d_L \tag{46}$$

$$e\frac{s_W}{3c_W}\bar{d}_R \gamma^\mu Z_\mu d_R \tag{47}$$

As before the couplings are given by

$$\frac{e}{s_W c_W}(T^3 - s_W^2 Q) \tag{48}$$

The charged currents will have the same structure as the leptonic currents. For instance:

$$-\frac{g}{\sqrt{2}}(\bar{u}_L \gamma^\mu W_\mu^+ d_L + hc) \tag{49}$$

We give the basic Feynman rules:

$W^\pm$ vertices:

$$ie\gamma^\mu(1 - \gamma^5)\frac{1}{2\sqrt{2}s_W} \tag{50}$$

Z vertices:

$$ie\gamma^\mu(v - a\gamma^5), \tag{51}$$

$$v = \frac{T^3 - 2s_W^2 Q}{2s_W c_W}, \qquad a = \frac{T^3}{2s_W c_W} \tag{52}$$

γ vertices:

$$-ieQ\gamma^\mu \tag{53}$$

Exercise.- Derive the above values for v and a.