12 Low-x region

The region below 10^{-2} had not been explored experimentally until very recently; a first look at these low-x values has been provided by the commissioning of HERA. HERA is a machine ideally suited for an in-depth analysis of structure functions. It should be possible to arrive at very low values of x (down to $x \sim 10^{-5}$).

Most parametrizations have traditionally performed very poorly when extrapolated to the low x region. Typically they predict an increase as $x \to 0$ which is lower than what is actually seen. The behaviour $F_2(x) \sim x^{-\lambda}$, with $\lambda \sim 1/2$ as $x \to 0$, which is predicted from the BFKL evolution equation seemed at some point to stand the comparison with HERA results best. However, this behaviour is still incompatible with unitarity and cannot hold all the way to $x = 0$ either. If fact we know now that the predictions from BFKL cannot be trusted. This has prompted a renewed interest in trying to extract the behaviour at low x from conventional Altarelli-Parisi evolution. The consensus now seems to be that even for the low values of x analyzed at HERA there is no real evidence of any results beyond ordinary perturbative QCD.

It is easy to understand why perturbative QCD must fail at some point. The expansion of the splitting function $P(z)$ in powers of α_s at the NLO actually resums all terms of the form $(\alpha_s \log Q^2)^n$ and $\alpha_s^n \log^{n-1} Q^2$. Looking at the propagator causing the mass singularity ($p = \xi P$)

$$\frac{1}{2pk} = -\frac{2x}{\xi k_T^2}. \tag{269}$$

Apart from the parametric integrals, we have

$$\int \frac{d^2 k_T}{k_T^2}. \tag{270}$$

This is the origin of the $\log \lambda^2$ and, eventually, of the $\log Q^2$.

The leading $\log^n Q^2$ will thus be produced by one single region in integration

$$\int \frac{d^2 k_T^n}{(k_T^n)^2} \int \frac{d^2 k_T^{n-1}}{(k_T^{n-1})^2} \ldots \int \frac{d^2 k_T^1}{(k_T^1)^2}, \tag{271}$$

with $|Q| \gg |k_T^n| \gg |k_T^{n-1}| \gg \ldots |k_T^1|$.

At sufficiently large x logarithms of $1/x$ necessarily appear. We have actually seen them in the double scaling limit. They must, at some point, spoil the predictivity of the perturbative expansion. One must then identify the regions in integration capable of giving rise to terms of the form $(\alpha_s \log \frac{1}{x})^n$, and eventually to $\alpha_s^n \log^{n-1} \frac{1}{x}$ and so on.

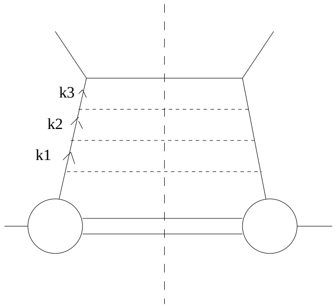

Figure 21: Ordering leading to the most singular $\log 1/x$ contribution.

Lipatov and coworkers have identified such a contribution. It corresponds to the diagram depicted in the above figure, more specifically to the region

$$k_i = \alpha_i P + \beta_i n + K_{iT}, \tag{272}$$

$$\alpha_1 \gg \alpha_2 \gg ... \gg \alpha_{n-1}, \qquad k_{iT} \sim k_{jT}, \qquad \beta_1 \ll \beta_2 \ll ... \ll \beta_{n-1}. \tag{273}$$

This leads to splitting kernels similar to those of the Altarelli-Parisi equations

$$F_2(x, Q^2) = \int d^2 k_T \int_x^1 \frac{d\xi}{\xi} C(\frac{x}{\xi}, Q^2, k_T) F_2(\xi, k_T), \tag{274}$$

where $F_2(\xi, k_T)$ obeys the differential equation

$$\xi \frac{\partial}{\partial \xi} F_2(\xi, k'_T) = \int d^2 k_T K(k'_T, k_T) \frac{{k'_T}^2}{k_T^2} F_2(\xi, k_T). \tag{275}$$

The BFKL kernel is now known to leading and subleading order.The leading asymptotic solution is

$$F(x, k_T) \sim x^{-4N \log \frac{2\alpha_s}{\pi}}. \tag{276}$$

Unfortunately the corrections implied by the next-to-leading calculations are gigantic. There is no way of doing anything useful with BFKL scaling at present. As previously discussed this does not seem to be a problem for HERA data since a careful analysis shows that —perhaps surprisingly— the data is well accounted for by ordinary perturbative QCD (the matter is however somewhat controversial to this date), but it will come the day where $\log 1/x$ corrections will be essential. The subject is thus still open.