2 Why QCD

Here we shall review some of the motivations that make us belive that QCD is the correct theory to describe the strong interactions. The presentation is not historical, but we emphasize some of the points that in hindsight force upon us the necessity for such a theory. The first evidence is the necessity of a new degree of freedom. There are three obvious experimental reasons for such a new degree of freedom that we review.

2.1 Quark model

Dwelling on the well known ideas of isospin symmetry, familiar to all of us, in the early 60's several physicists (Han, Nambu and, most notably, Gell-Mann and Zweig) explored the possibility of extending $SU(2)$ isospin symmetry to a larger group. The obvious choice was of course $SU(3)$. If the isospin paradigm is to work here too, particles —i.e. physical states— subject to strong interactions would order themselves following the irreps of $SU(3)$ exactly as they follow the irreps of $SU(2)$ (or the irreps of the angular momentum for that matter, in the case of particles with half-integer spin, which is also $SU(2)$).
The irreps of $SU(3)$ are more complicated than those of $SU(3)$. Details can be found in Cheng and Li, for instance. I encourage you to work out in detail the following instructions and exercise

Exercise.- Irreps of $SU(3)$ (borrowed from a course by H.P. Nilles):
http://www.ecm.ub.es/∼espriu/web/irrepsSU3.ps . For further guidance see e.g.: H. Georgi, *Lie algebras in particle physics*, Benjamin-Cummings (1982).

There are representations of dimension **3**, **6**, **8**, **10**, etc. A given representation and its conjugate are not equivalent, in general, unlike in $SU(2)$; that is they cannot be made to coincide by a change of basis.
The crucial observation of Gell-Mann and others was that all known particles fall into multiplets of $SU(3)$. Roughly speaking all lightest mesons and baryons would fall into octets or decuplets of $SU(3)$, exactly as all hadrons and nuclei fall into multiplets of $SU(2)$ of isospin. We thus have for instance an octet of pseudoscalar mesons ($J^P = 0^-$) composed by two isospin doublets (K^0, K^+ and $K^-, \bar{K}^0$), one isospin triplet (π^-, π^0, π^+) and one isospin singlet (η),

$$\begin{array}{ccccc} & K^0 & & K^+ & \\ \pi^- & & \pi^0, \eta & & \pi^+ \\ & K^- & & \bar{K}^0 & \end{array} \tag{36}$$

a $1/2^+$ "nucleon" octet $(n, p, \Xi^-, \Xi^0, \Sigma^\pm, \Sigma^0, \Lambda)$ with a similar structure, a $3/2^+$ baryon decuplet

$$\begin{array}{ccccccc} \Delta^- & & \Delta^0 & & \Delta^+ & & \Delta^{++} \\ & \Sigma^{*-} & & \Sigma^{*0} & & \Sigma^{*+} & \\ & & \Xi^{*-} & & \Xi^{*0} & & \\ & & & \Omega^- & & & \end{array} \quad (37)$$

etc. Note that since $SU(2)$ is a subgroup of $SU(3)$, the components of the representations of the latter decompose into isospin representations.

It turns out that hadronic states in nature appear only in certain representations of $SU(3)$ and others are never observed. For instance, no single hadron has ever been found in a **6** or in the fundamental **3** or $\bar{\mathbf{3}}$ representation of $SU(3)$. This is quite remarkable as all other representations can br constructed by direct products of this one (like one can construct representations of arbitrary high angular momentum by composing $J = 1/2$ representations). The hypothesis that it was put forward in the 60's is that the states in the fundamental representation correspond to some hypothetical (at the time) particle, the "quark", that transformed as a vector under $SU(3)$ —the fundamental representation **3**. Their three components were labelled

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad (38)$$

The conjugate representation corresponds to the contravariant vector. All hadrons are then "formed" by composing the fundamental representation. But is this real or just a mathematical curiosity? Why aren't quarks observed?

The charge assignment of the u-type quarks can be inferred from isospin and the charge of the nucleons, $q_u = 2/3$, $q_d = -1/3$. The one of the s-type quark has to be so as to match the charges of the observed hadrons; it can be seen to be $q_s = -1/3$).

Taking a look at the decuplet we pay attention to the Δ^{++} $3/2^+$ state. According to the above principle, it can be understood as being "formed" of three quarks of the same type u. This immediately raises a problem as, from angular momentum considerations $|\Delta^{++}\rangle = |u^\uparrow u^\uparrow u^\uparrow\rangle$. Being the lightest hadron with this quark contents we expect to have the three quarks in the ground state, hence in a symmetric wave function. This is in contradiction with Fermi statistics. The contradiction can be solved if we admit the existence of a new quantum number α and

$$|u^\uparrow u^\uparrow u^\uparrow\rangle = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|u^\uparrow_\alpha u^\uparrow_\beta u^\uparrow_\gamma\rangle. \tag{39}$$

This new quantum number is called color.

2.2 $\sigma(e^+e^- \to hadrons)$

If we accept the possibility of the "normal" hadrons being composed by quarks, why aren't they seen? Perhaps ther are bound by forces that are strong enough to confine them permanetly into hadrons. But if this is the case, by probing a hadron with a large enough momentum transfer we should "see" somehow the quarks. If we believe this, for large momentum transfer, and looking for fully inclusive cross-sections, it is natural to expect that

$$R_{had} = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \sum_{i=1}^{n} q_i^2, \tag{40}$$

because $\sigma(e^+e^- \to hadrons) \simeq \sigma(e^+e^- \to \bar{q}q)$ if at large enough momentum transfer (= short distances) quarks are indeed free The index i runs over all degrees of freedom coupling to the intermediate photon (with charge q_i).

$$n = 3 \qquad u, d, s \qquad\qquad R = \frac{2}{3} \tag{41}$$

$$n = 4 \qquad u, d, s, c \qquad\qquad R = \frac{10}{9} \tag{42}$$

$$n = 6 \qquad u, d, s, c, b, t \qquad R = \frac{5}{3}. \tag{43}$$

To consider $n = 3, 4$ or 5 depends on the number of quarks that are "active", i.e. those we can actually produce with the available energy. This should obviously be at least greater than (twice) their rest mass, namely $m_u \sim m_d \simeq 10$ MeV, $m_s \simeq 100$ MeV, $m_c \simeq 1.5$ GeV, $m_b \simeq 4.5$ GeV and $m_t \simeq 170$ GeV. Experimentally all the values in (41-42) are off by about a factor 3. The quark model can again be reconciled with experiment if we admit that the new quantum number can take 3 values and is blind to electromagnetic interactions, q^α, $\alpha = 1, 2, 3$.

A famous series of experiments, called deep inelastic scattering, clearly showed 30 years ago the existence of hard processes inside nucleons, at very short distances. One can get a rough picture of these processes by just assuming that quarks are approximately free if one looks at distances $\ll 1$ (GeV)$^{-1}$. One needs a rather peculiar type of theory. It must be a theory whose elementary fields are fermions (quarks), but weakly interacting at short distances. In addition we would like it to be a renormalizable theory to be able to apply the full machinery Quantum Field Theory and this pretty much forces us to work with gauge theories. Trivial modifications of QED will not work because we need anti-screenning of charges rather than screening. We will see in a while that the appropriate modification is provided by going to a non-abelian gauge theory.

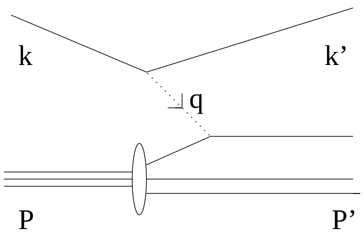

Figure 1: The kinematics of deep inelastic scattering.

2.3 The decay $\pi^0 \to \gamma\gamma$

The other process relevant at this point is the celebrated $\pi^0 \to \gamma\gamma$ decay. This process goes dominantly through the triangle diagram with a closed loop of quarks. This diagram is also called 'the anomaly', for reasons that we will discuss in a while. The calculation gives

$$\Gamma = \frac{1}{576\pi^3} \frac{\alpha^2}{f_\pi^2} m_\pi^3 = 0.85\text{eV}. \tag{44}$$

Experimentally $\Gamma = 7.37 \pm 0.5$ eV. The result is off by a factor $9 = 3^2$ which, again, is understood if we accept that additional degrees of freedom, invisible to both photon and pion, exist. So we learn that ordinary hadrons seem not to carry any color. Similar analysis show that color does not couple to the W or Z bosons either.

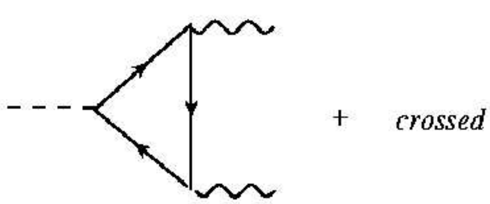

Figure 2: The triangle diagram.

Mimicking the most successful quantum theory up to this date (and certainly the only one known at the beginning of the 70's), let's introduce a gauge symmetry acting on the new degree of freedom just as Electromagnetism acts on the electric charge with a gauge group $U(1)$. It must be a new gauge group, since color does not couple to known gauge fields. We want a group with irreducible representations of dimension 3, for the quark model to fit in. Obvious candidates are $O(3)$, $SU(2)$, $U(2)$, $U(3)$, $SU(3)$, $SO(3)$ and $Sp(2)$. Groups such as $U(2)$, $U(3)$ and $O(3)$ can be discarded right away because $\epsilon^{\alpha\beta\gamma}$ is not an invariant tensor (and we need that for the Δ^{++}). On the other hand $Sp(2) \simeq SO(3) = SU(2)/Z_2$. Neither of these have *complex* representations of dimension 3 (hence they would lead to diquark states). We are left with $SU(3)$.

Historically, non-abelian gauge theories were introduced by Yang and Mills in a completely different concept in 1954. They proposed a non-abelian gauge symemtry where rotations were made in the space of isotopic spin; i.e. they proposed raising isosping symmetry to the status of gauge invariance. This did not quite work at the moment because no one knew how to give masses to the "photons", but it is not too far from the currently accepted (and well tested) electroweak theory.