Standard Model

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Exercise 7

Check the counterterm structure that makes finite the Green function $\langle TWWW\rangle$ in QCD.

Redefining the coupling constant and the gluon fields as

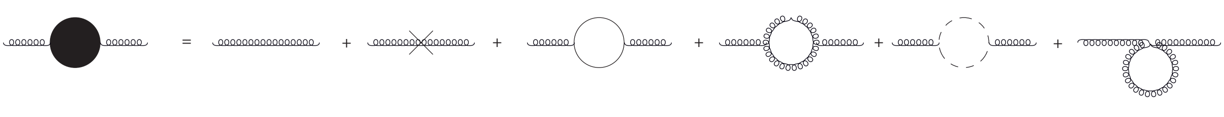
$$g_0 = Z_g g = (1 + dZ_g)g$$

$$W^0_\mu = Z^{\frac{1}{2}}_{3YM} W_\mu = (1 + \frac{1}{2}dZ_{3YM})W_\mu,$$

the gluon part of the bare QCD lagrangian is

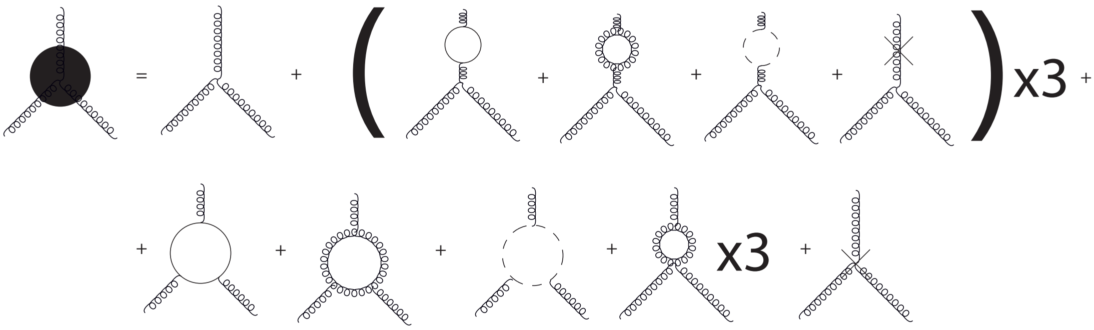
$$\begin{aligned}
L = &-\frac{1}{4}(\partial_\mu W^{0a}_\nu - \partial_\nu W^{0a}_\mu + g_0 f_{abc} W^{b0}_\mu W^{c0}_\nu)(\partial^\mu W^{\nu 0a} - \partial^\nu W^{\mu 0a} + g_0 f_{ade} W^{\mu d0} W^{\nu e0}) = \\
= &-\frac{1}{4}\Big[(\partial_\mu W^{0a}_\nu - \partial_\nu W^{0a}_\mu)(\partial^\mu W^{\nu 0a} - \partial^\nu W^{\mu 0a}) + 2g_0 f_{ade}(\partial_\mu W^{0a}_\nu - \partial_\nu W^{0a}_\mu)(W^{\mu d0} W^{\nu e0}) + \\
&+ g_0^2 f_{abc} f_{ade} W^{b0}_\mu W^{c0}_\nu W^{\mu d0} W^{\nu e0}\Big] = \\
= &-\frac{1}{4}\Big[(1 + \frac{1}{2}dZ_{3YM})^2(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{\nu a} - \partial^\nu W^{\mu a}) + \\
&+ 2(1 + \frac{1}{2}dZ_{3YM})^3(1 + dZ_g) g f_{ade}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(W^{\mu d} W^{\nu e}) + \\
&+ (1 + \frac{1}{2}dZ_{3YM})^4(1 + dZ_g)^2 g^2 f_{abc} f_{ade} W^b_\mu W^c_\nu W^{\mu d} W^{\nu e}\Big] = \\
= &-\frac{1}{4}\Big[(1 + dZ_{3YM})(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{\nu a} - \partial^\nu W^{\mu a}) + \\
&+ 2(1 + \frac{3}{2}dZ_{3YM})(1 + dZ_g) g f_{ade}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(W^{\mu d} W^{\nu e}) + \\
&+ (1 + 2dZ_{3YM})(1 + 2dZ_g) g^2 f_{abc} f_{ade} W^b_\mu W^c_\nu W^{\mu d} W^{\nu e}\Big] \simeq \\
\simeq &-\frac{1}{4}\Big[(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{\nu a} - \partial^\nu W^{\mu a}) + 2g f_{ade}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(W^{\mu d} W^{\nu e}) + \\
&+ g^2 f_{abc} f_{ade} W^b_\mu W^c_\nu W^{\mu d} W^{\nu e} + dZ_{3YM}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{\nu a} - \partial^\nu W^{\mu a}) + \\
&+ 2(dZ_g + \frac{3}{2}dZ_{3YM}) g f_{ade}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(W^{\mu d} W^{\nu e}) + 2(dZ_g + dZ_{3YM}) g^2 f_{abc} f_{ade} W^b_\mu W^c_\nu W^{\mu d} W^{\nu e}\Big].
\end{aligned}$$

The first term gives us, through the Euler-Lagrange equations, the feynman rule for the propagator. The fourth term will give us the same propagator but with a factor dependent of dZ_{3YM}. Renormalizing at one loop, dZ_{3YM} is determined by making finite the Green function $\langle TWW\rangle$. Considering the interaction terms in the lagrangian and their corresponding vertices and propagators, this green function is composed of the five following diagrams,



The propagator with a cross corresponds to the fourth therm in the lagrangian. The last diagram is zero. The three other loops are proportional to the gluon propagator, and their factors diverge. Schematically, for $\langle TWW \rangle$,

We see then that the task of dZ_{3YM} is to cancel the loops in the 2-gluon green functions and is thus determined. The role of dZ_g is to make finite $\langle TWWW \rangle$. Its diagrams are



The procedure is the same. The diagrams with the cross in a leg already take care of the ones with a loop in the same leg (due to the chosen Z_{3YM}). The divergent parts of the other loops, therefore, must be cancel·led by the last diagram, corresponding to the sixth term in lagrangian above. Thus dZ_g is determined. Instead of working with Z_g we can define $Z_{1YM} \equiv Z_g Z_{3YM}^{3/2}$. This way,

$$1 + dZ_{1YM} = (1 + dZ_g)(1 + Z_{3YM})^{\frac{3}{2}} \simeq 1 + dZ_g + \frac{3}{2} dZ_{3YM}$$

$$\Rightarrow \quad dZ_{1YM} \simeq dZ_g + \frac{3}{2} dZ_{3YM}.$$

Then, the corresponding interaction term in the lagrangian above is

$$2dZ_{1YM} g f_{ade} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)(W^{\mu d} W^{\nu e}).$$