Ejercicios de Standard Model

[1] Ejercicio 8:

Local symmetries: Try to repeat the arguments leading to the derivation of a Noether current (4), but now considering ϵ_a *as a function of x, i.e.* ϵ_a. *Obviously we cannot factor out* ϵ_a *now. Check that if we replace* ∂_μ *by* D_μ *with a gauge connection we can remove the offending term by choosing an appropiate transformation for gauge field.*

Asumimos una transformacion (local)

$$\delta\Psi = -i\epsilon_a(x)T^a\Psi \tag{1}$$

Como

$$\partial_\mu(\delta\Psi) = -i(\partial_\mu\epsilon_a(x))T^a\Psi - i\epsilon_a(x)T^a\partial_\mu\Psi \tag{2}$$

por lo tanto se observa que $\partial_\mu(\delta\Psi) \neq \delta(\partial_\mu\Psi)$ Entonces como

$$\delta L = \frac{\delta L}{\delta\Psi}\delta\Psi + \frac{\delta L}{\delta(\partial_\mu\Psi)}\delta(\partial_\mu\Psi) \tag{3}$$

Sabemos que las ecuaciones de Euler Lagrange

$$\partial_\mu\frac{\delta L}{\delta(\partial_\mu\Psi)} = \frac{\delta L}{\delta\Psi} \tag{4}$$

que al reemplazar en (3)

$$\delta L = \partial_\mu\frac{\delta L}{\delta(\partial_\mu\Psi)}\delta\Psi + \frac{\delta L}{\delta(\partial_\mu\Psi)}\delta(\partial_\mu\Psi) \tag{5}$$

$$\delta L = \partial_\mu\frac{\delta L}{\delta(\partial_\mu\Psi)}\delta\Psi + \frac{\delta L}{\delta(\partial_\mu\Psi)}\partial_\mu(\delta\Psi) + i\frac{\delta L}{\delta(\partial_\mu\Psi)}(\partial_\mu\epsilon_a(x))T^a\Psi \tag{6}$$

$$\delta L = \partial_\mu\left(\frac{\delta L}{\delta(\partial_\mu\Psi)}\delta\Psi\right) - i\frac{\delta L}{\delta(\partial_\mu\Psi)}(-i\partial_\mu\epsilon_a(x)T^a\Psi) \tag{7}$$

El ultimo termino es $\delta\Psi$. Por lo tanto no se puede definir J^μ tal que $\partial_\mu J^\mu = 0$ excepto si se reemplaza $\partial_\mu \to D_\mu$. Y ahora $A'_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu\epsilon_a(x)$ entonces vemos que $D_\mu J^\mu = 0$ donde $J_\mu = \frac{\delta L}{\delta(\partial_\mu\Psi)}\delta\Psi$.

Ejercicio 9:

Determine the transformation of D_μ *and* $F_{\mu\nu}$ *in a non-abelian gauge theory under gauge transformation G(x).*

---

[1] Nota: las ecuaciones () se refieren a las de estas soluciones y [] a las del resumen del curso.

Como sabemos

$$L = i\bar{\Psi}\gamma^{\mu}(\partial_{\mu} + gA_{\mu}T)\Psi \tag{8}$$

y queremos que cumpla con ser invariante al transformarlo (recordar que $D_{\mu} = \partial_{\mu} + gA_{\mu}T$)

$$L' = i\bar{\Psi}\gamma'^{\mu}D'_{\mu}\Psi' \tag{9}$$

Entonces

$$L = i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + g\bar{\Psi}\gamma^{\mu}A_{\mu}\hat{T}\Psi \tag{10}$$

$$L = i\bar{\Psi}\hat{G}^{-1}\hat{G}\gamma^{\mu}\partial_{\mu}\hat{G}^{-1}\hat{G}\Psi + g\bar{\Psi}\hat{G}^{-1}\hat{G}\gamma^{\mu}A_{\mu}\hat{T}\hat{G}^{-1}\hat{G}\Psi \tag{11}$$

Y como $\bar{\Psi}' = \bar{\Psi}\hat{G}^{-1}$ y $\Psi' = \hat{G}\Psi$

$$L = i\bar{\Psi}'\hat{G}\gamma^{\mu}\partial_{\mu}\hat{G}^{-1}\Psi' + g\bar{\Psi}'\hat{G}\gamma^{\mu}A_{\mu}\hat{T}\hat{G}^{-1}\Psi' \tag{12}$$

La $\hat{G}$ conmuta con las gamma y además sumo y resto un termino $i\bar{\Psi}'\gamma^{\mu}\partial_{\mu}\Psi'$ y reagrupando

$$L = i\bar{\Psi}'\gamma^{\mu}\partial_{\mu}\Psi' + i\bar{\Psi}'\gamma^{\mu}(\hat{G}(\partial_{\mu}\hat{G}^{-1}))\Psi' + g\bar{\Psi}'\gamma^{\mu}\hat{G}A_{\mu}\hat{T}\hat{G}^{-1}\Psi' \tag{13}$$

$$L = i\bar{\Psi}'\gamma^{\mu}\partial_{\mu}\Psi' + g\bar{\Psi}'\gamma^{\mu}\left(\frac{i}{g}\hat{G}(\partial_{\mu}\hat{G}^{-1}) + \hat{G}A_{\mu}\hat{T}\hat{G}^{-1}\right)\Psi' \tag{14}$$

Con lo que

$$\hat{A}'_{\mu} = \frac{i}{g}\hat{G}(\partial_{\mu}\hat{G}^{-1}) + \hat{G}\hat{A}_{\mu}\hat{G}^{-1} \tag{15}$$

Para que D_{μ} sea covariante

$$\hat{D}_{\mu} \rightarrow \hat{D}'_{\mu} = \partial_{\mu} - ig\hat{A}'_{\mu}\hat{T} \tag{16}$$

Por lo tanto

$$\hat{D}'_{\mu} = \hat{G}\hat{D}_{\mu}\hat{G}^{-1} \tag{17}$$

Ademas sabemos que se puede escribir al tensor de energia impulso como

$$\hat{F}_{\mu\nu} = \frac{i}{g}[\hat{D}_{\mu}, \hat{D}_{\nu}] \tag{18}$$

que por (17) como

$$[\hat{D}'_{\mu}, \hat{D}'_{\nu}] = \hat{G}[\hat{D}_{\mu}, \hat{D}_{\nu}]\hat{G}^{-1} \tag{19}$$

entonces

$$\hat{F}'_{\mu\nu} = \hat{G}\hat{F}_{\mu\nu}\hat{G}^{-1} \tag{20}$$

Ejercicio 10:
Verify that the Feynman rule for the three-gluon vertex complies with Bose symmetry.
La regla de Feynman es

$$igf_{rst}[(k_1^\tau - k_2^\tau)g^{\rho\sigma} + (k_2^\rho - k_3^\rho)g^{\sigma\tau} + (k_3^\sigma - k_1^\sigma)g^{\tau\rho}](2\pi)^4\delta^4(k_1 + k_2 + k_3) \tag{21}$$

Al cambiar por ejemplo $(k_1, r, \rho) \leftrightarrow (k_2, s, \sigma)$

$$igf_{srt}[(k_2^\tau - k_1^\tau)g^{\sigma\rho} + (k_1^\sigma - k_3^\sigma)g^{\rho\tau} + (k_3^\rho - k_2^\rho)g^{\tau\sigma}](2\pi)^4\delta^4(k_2 + k_1 + k_3) \tag{22}$$

Como

$$f_{srt} = -f_{rst} \tag{23}$$

y ademas

$$g^{\sigma\rho} = g^{\rho\sigma} \tag{24}$$

y lo mismo para $g^{\rho\tau} = g^{\tau\rho}$ y $g^{\tau\sigma} = g^{\sigma\tau}$ entonces

$$ig(-f_{rst})[-(k_1^\tau - k_2^\tau)g^{\rho\sigma} - (k_2^\rho - k_3^\rho)g^{\sigma\tau} - (k_3^\sigma - k_1^\sigma)g^{\tau\rho}](2\pi)^4\delta^4(k_2 + k_1 + k_3) \tag{25}$$

que es equivalente a (21) por lo tanto cumple cpn la simetria de Bose.

Ejercicio 11:
Find the sum over physical states (i.e. over transverse states in the case of massless gauge particles) of the polarization vectors $\tau^{\mu\nu} = \sum_\sigma \epsilon^\mu(k,\sigma)\epsilon^{*\nu}(k,\sigma)$

Queremos encontrar

$$\sum_{\sigma=0}^{3} g_{\sigma\sigma}\epsilon_\mu(k,\sigma)\epsilon_\nu(k,\sigma) \tag{26}$$

donde $g_{\sigma\sigma}$ es el factor signo que valdra 1 o -1.
Introducimos dos vectopres transversales $\sigma = 1, 2$

$$\begin{aligned} \epsilon_\mu(k,1) &= (0, \epsilon(k,1)) \\ \epsilon_\mu(k,2) &= (0, \epsilon(k,2)) \end{aligned} \tag{27}$$

respecto a un sistema de Lorentz fijo. Como no podemos utilizar k como un vector d ela base (ya que no es normalizable) definimos un vector en el sistema de referencia de Lorentz dado por

$$n = (1, 0, 0, 0) \tag{28}$$

donde $n^2 = +1$.
El vector polarizacion longitudinal se puede escribir

$$\epsilon(k,3) = \frac{k - n(k.n)}{\sqrt{(k.n)^2 - k^2}} \tag{29}$$

cuya norma es la correcta $\epsilon(k,3).\epsilon(k,3) = -1$. Ademas $\epsilon(k,3).n = 0$ asi que por la condicion de n normalizado vemos que

$$\epsilon(k,3) = (0, \frac{k}{|k|^2}) \tag{30}$$

Notemos que $\epsilon(k,i)$ con $i = 1,2,3$ tienen componentes espaciales ortogonales.
El cuarto vector sera ortogonal a los tres

$$\epsilon(k,0) = n \tag{31}$$

cuya norma es +1
Asi vemos que

$$\begin{aligned} k^\mu.\epsilon_\mu(k,1) = k^\mu.\epsilon_\mu(k,2) = 0 \\ k^\mu.\epsilon_\mu(k,0) = -k^\mu.\epsilon_\mu(k,3) = k.n \end{aligned} \tag{32}$$

Haremos un parentesis donde conjeturaremos que la ecuacion (26) es

$$\sum_{\sigma=0}^{3} g_{\sigma\sigma}\epsilon_\mu(k,\sigma)\epsilon_\nu(k,\sigma) = g_{\mu\nu} \tag{33}$$

Cuando $\mu = 0$ y $\nu = 0$ el resultado es $+1 = g_{00}$ por lo que queda demostrar cuando los indices son espaciales i,j (ya que los indices o componentes que son mezcla espacio temporales, se anulan)

$$\sum_{\sigma=1}^{3} \epsilon_i(k,\sigma)\epsilon_j(k,\sigma) = \delta_{ij} \tag{34}$$

que es la relacion de completitud ordinaria. Lo que se queria demostrar.
Retomando:
La suma entonces sobr elos estados transversales sera

$$-\sum_{\sigma=1}^{3} \epsilon_\mu(k,\sigma)\epsilon_\nu(k,\sigma) = g_{\mu\nu} + \frac{k_\mu.k_\nu}{(k.n)^2} + \epsilon_\mu(k,3)\epsilon_\nu(k,3) \tag{35}$$

y como

$$\epsilon_\mu(k,3)\epsilon_\nu(k,3) = -\frac{k_\mu.n_\nu + k_\nu.n_\mu}{(k.n)} \tag{36}$$

entonces

$$\sum_{\sigma=1}^{2} \epsilon_\mu(k,\sigma)\epsilon_\nu(k,\sigma) = -g_{\mu\nu} - \frac{k_\mu.k_\nu}{(k.n)^2} + \frac{k_\mu.n_\nu + k_\nu.n_\mu}{(k.n)} \tag{37}$$

Ejercicio 12:
Verify that $\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta}$ *is the divergence of the term* $\epsilon^{\mu\nu\alpha\beta}(W^a_\nu\partial_\alpha W^a_\beta + \frac{2g}{3}f_{abc}W^a_\nu W^b_\alpha W^c_\beta)$

Quiero demostrar que

$$\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta_a} = \partial_\mu[\epsilon^{\mu\nu\alpha\beta}(W^a_\nu\partial_\alpha W^a_\beta + \frac{2g}{3}f_{abc}W^a_\nu W^b_\alpha W^c_\beta)] \tag{38}$$

Veamos la l.h.s.

$$\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta_a} = \epsilon^{\mu\nu\alpha\beta}(\partial_\mu W^a_\nu + gf^a_{bc}W^b_\mu W^c_\nu)(\partial_\alpha W_{\beta_a} + gf_{ade}W^d_\alpha W^e_\beta) \tag{39}$$

$$\begin{aligned}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta_a} = \epsilon^{\mu\nu\alpha\beta}(&\partial_\mu W^a_\nu\partial_\alpha W_{\beta_a} + gf^a_{bc}\partial_\alpha W_{\beta_a}W^b_\mu W^c_\nu + \\ &gf_{ade}\partial_\mu W^a_\nu W^d_\alpha W^e_\beta + g^2 f^a_{bc}f_{ade}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta)\end{aligned} \tag{40}$$

El tercer termino cuando b=d,c=e

$$\epsilon^{\mu\nu\alpha\beta}gf^a_{bc}\partial_\alpha W_{\beta_a}W^b_\mu W^c_\nu = \epsilon^{\mu\nu\alpha\beta}gf_{ade}\partial_\alpha W_\beta{}^a W^d_\mu W^e_\nu \tag{41}$$

al intercambiar $\mu \leftrightarrow \alpha$

$$\epsilon^{\mu\nu\alpha\beta}gf^a_{bc}\partial_\alpha W_{\beta_a}W^b_\mu W^c_\nu = -\epsilon^{\mu\nu\alpha\beta}gf_{ade}\partial_\mu W_\beta{}^a W^d_\alpha W^e_\nu \tag{42}$$

al intercambiar $\nu \leftrightarrow \beta$

$$\epsilon^{\mu\nu\alpha\beta}gf^a_{bc}\partial_\alpha W_{\beta_a}W^b_\mu W^c_\nu = +\epsilon^{\mu\nu\alpha\beta}gf_{ade}\partial_\mu W_\nu{}^a W^d_\alpha W^e_\beta \tag{43}$$

El cuarto termino

$$\epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta = \epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^b_{[\mu}W^c_\nu W^d_\alpha W^e_{\beta]} \tag{44}$$

tambien los indices latinos

$$\epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta = \epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^{[b}_{[\mu}W^c_\nu W^d_\alpha W^{e]}_{\beta]} \tag{45}$$

o bien

$$\epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta = \epsilon^{\mu\nu\alpha\beta}g^2 f_{a[de}f^a_{bc]}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta \tag{46}$$

la identidad de Jacobi es $f_{a[de}f^a_{bc]} = 0$

$$\epsilon^{\mu\nu\alpha\beta}g^2 f^a_{bc}f_{ade}W^b_\mu W^c_\nu W^d_\alpha W^e_\beta = 0 \tag{47}$$

Entonces el l.h.s. se reduce a

$$\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta_a} = \epsilon^{\mu\nu\alpha\beta}(\partial_\mu W^a_\nu\partial_\alpha W_{\beta_a} + 2gf_{ade}\partial_\mu W^a_\nu W^d_\alpha W^e_\beta) \tag{48}$$

El r.h.s. de la igualdad

$$\begin{aligned}\partial_\mu[\epsilon^{\mu\nu\alpha\beta}(W^a_\nu\partial_\alpha W^a_\beta + \frac{2g}{3}f_{abc}W^a_\nu W^b_\alpha W^c_\beta)] = \epsilon^{\mu\nu\alpha\beta}[&\partial_\mu W^a_\nu\partial_\alpha W^a_\beta + W^a_\nu\partial_\mu\partial_\alpha W^a_\beta + \\ &\frac{2g}{3}f_{abc}\left(\partial_\mu W^a_\nu W^b_\alpha W^c_\beta + W^a_\nu\partial_\mu W^b_\alpha W^c_\beta + W^a_\nu W^b_\alpha\partial_\mu W^c_\beta\right)]\end{aligned} \tag{49}$$

ya que $\epsilon^{\mu\nu\alpha\beta}$ es antisimetrico y $\partial_\mu \partial_n u$ simetrico el segundo termino se anula. Por otro lado los ultimos dos terminos al intercambiar $\alpha \leftrightarrow \mu$ y $a \leftrightarrow b$

$$\epsilon^{\mu\nu\alpha\beta} f_{abc} W^a_\nu \partial_\mu W^b_\alpha W^c_\beta = \epsilon^{\mu\nu\alpha\beta} f_{abc} W^b_\alpha \partial_\mu W^a_\nu W^c_\beta \tag{50}$$

y intercambiando $\nu \leftrightarrow \beta$ y $a \leftrightarrow c$

$$\epsilon^{\mu\nu\alpha\beta} f_{abc} W^a_\nu W^b_\alpha \partial_\mu W^c_\beta = \epsilon^{\mu\nu\alpha\beta} f_{abc} W^c_\beta W^b_\alpha \partial_\mu W^a_\nu \tag{51}$$

Entonces

$$\begin{aligned} \partial_\mu [\epsilon^{\mu\nu\alpha\beta} (W^a_\nu \partial_\alpha W^a_\beta + \frac{2g}{3} f_{abc} W^a_\nu W^b_\alpha W^c_\beta)] = \\ \epsilon^{\mu\nu\alpha\beta} [\partial_\mu W^a_\nu \partial_\alpha W^a_\beta + 2 f_{abc} \partial_\mu W^a_\nu W^b_\alpha W^c_\beta] \end{aligned} \tag{52}$$

y vemos que es igual a (48) como se queria demostrar.

Ejercicio 13:
Would it be possible to have $Q^a|0\rangle \neq 0$*?*

Como

$$-iT^a v = \langle 0|[Q^a, \phi]|0\rangle \tag{53}$$

Cuando $Q^a|0\rangle \neq 0$ no implica que $\langle 0|[Q^a, \phi]|0\rangle \neq 0$. De hecho basta con que $[Q^a, \phi] = 0$ para que entonces $T^a v = 0$. Por lo tanto si es posible tener $Q^a|0\rangle \neq 0$ pero $T^a v = 0$

Ejercicio 14:
Discuss the Goldstone phenomenon in an Ising ferromagnet in 3 dimensions. What is $Q|0\rangle$ *in this case?*

Ejercicio 15:
Derive eq. (101)

Ejercicio 16:
Show that the coefficient of the anomaly above is correct.

Sabemos por la ecuacion [77] de los apuntes que

$$\partial_\mu (\bar{\Psi} \gamma^\mu \gamma^5 \Psi) = \frac{g^2}{4\pi^2} \frac{N_f}{8} \epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F_{\alpha\beta a} \tag{54}$$

Habiamos definido la corriente axial como

$$J^\mu_5 = \bar{\Psi} \gamma^\mu \gamma^5 \Psi \tag{55}$$

y por otro lado la constante

$$\alpha = \frac{g^2}{4\pi} \tag{56}$$

donde $g \equiv e$ en la constante de estructura fina $\alpha = \frac{e^2}{4\pi} = \frac{1}{127}$. Esto hace que (27) sea

$$\partial_\mu J_5^\mu = \frac{\alpha}{\pi}\frac{N_f}{8}\left(\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta a}\right) \tag{57}$$

Pero como

$$\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F_{\alpha\beta a} = F^a_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta a} = 2F^a_{\mu\nu}\tilde{F}_{\mu\nu a} \tag{58}$$

porque $\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta a} \equiv 2\tilde{F}_{\mu\nu a}$ Entonces

$$\partial_\mu J_5^\mu = \frac{\alpha}{4\pi}N_f\left(F^a_{\mu\nu}\tilde{F}_{\mu\nu a}\right) \tag{59}$$

Ejercicio 17:
Compute the anomaly in the two space-time dimensions.

Ejercicio 18:
Check the counterterm structure that makes finite the Green function $\Gamma = \langle TWWW\rangle$. *You have to renormalize external legs and the charge.*

Como llos valores bare son $g^0 = Z_{1YM}Z_{3YM}^{-\frac{3}{2}}g$ y $W^0_\mu = Z_{3YM}^{\frac{1}{2}}W_\mu$

$$\langle 0|TW^0_\mu W^0_\mu W^0_\mu|0\rangle = Z_{1YM}Z_{3YM}^{-\frac{3}{2}}(Z_{3YM}^{\frac{1}{2}})^3\langle 0|TW_\mu W_\mu W_\mu|0\rangle \tag{60}$$

Y asi

$$\langle 0|TW^0_\mu W^0_\mu W^0_\mu|0\rangle = Z_{1YM}\langle 0|TW_\mu W_\mu W_\mu|0\rangle \tag{61}$$

Ejercicio 19:
Verify (133) is indeed the solution of the differential equation (132)

Veamos si llamamos $U = tD - \int_0^t dt\gamma_\Gamma(\bar{\alpha}_s(t))$ y $V = \Gamma(p_i, \bar{\alpha}_s(t), \mu)$

$$\frac{\partial\Gamma}{\partial t} = (D - \gamma_\Gamma(\bar{\alpha}_s(t)))(expU)V + (expU)\frac{\partial V}{\partial\bar{\alpha}}\frac{\partial\bar{\alpha}}{\partial\alpha}\frac{\partial\alpha}{\partial t} \tag{62}$$

$$\frac{\partial\Gamma}{\partial\alpha_s} = \left(\int_0^t dt\frac{\partial\gamma_\Gamma}{\partial\bar{\alpha}}\frac{\partial\bar{\alpha}}{\partial\alpha}\right)(expU)V + (expU)\frac{\partial V}{\partial\bar{\alpha}}\frac{\partial\bar{\alpha}}{\partial\alpha} \tag{63}$$

Como la dimension anomala γ_Γ es independiente de $\bar{\alpha}_s$ ya que $\gamma_\Gamma \sim O(Z_\Gamma)$ y $Z_\Gamma(\mu,\epsilon) \to \frac{\partial\gamma_\Gamma}{\partial\bar{\alpha}_s} = 0$ por lo tanto el primer termino del r.h.s. se anula. Entonces

$$\begin{gathered}\left(-\frac{\partial\Gamma}{\partial t} + \beta\alpha_s\frac{\partial\Gamma}{\partial\alpha_s} - \gamma_\Gamma + D\right)\Gamma(e^t p_i, \bar{\alpha}_s(t), \mu) = \\ -D\Gamma + \gamma_\Gamma(\bar{\alpha}_s)\Gamma - (expU)\frac{\partial V}{\partial\bar{\alpha}}\frac{\partial\bar{\alpha}}{\partial\alpha}\frac{\partial\alpha}{\partial t} + \beta\alpha(expU)\frac{\partial V}{\partial\bar{\alpha}}\frac{\partial\bar{\alpha}}{\partial\alpha} + D\Gamma - \gamma_\Gamma(\bar{\alpha}_s)\Gamma\end{gathered} \tag{64}$$

Vemos que el primero y segundo termino se simplifican con los dos ultimos. Finalmente $\bar{\alpha}(t) \to \alpha(e^t \mu)$ entonces $\frac{\partial \alpha}{\partial t} = \frac{\partial \alpha}{\partial (e^t \mu)} \frac{\partial (e^t \mu)}{\partial t} = \frac{\partial \alpha}{\partial \mu} \mu = \alpha_s \beta$ (la ultima igualdad corresponde a la ecuacio [134]) y

$$\left(-\frac{\partial \Gamma}{\partial t} + \beta \alpha_s \frac{\partial \Gamma}{\partial \alpha_s} - \gamma_\Gamma + D \right) \Gamma(e^t p_i, \bar{\alpha}_s(t), \mu) = \\ -(expU) \frac{\partial V}{\partial \bar{\alpha}} \frac{\partial \bar{\alpha}}{\partial \alpha} \alpha_s \beta + \beta \alpha (expU) \frac{\partial V}{\partial \bar{\alpha}} \frac{\partial \bar{\alpha}}{\partial \alpha} = 0 \tag{65}$$

Lo que se queria demostrar.

Ejercicio 20:
If

$$Z_\alpha = 1 + \frac{\alpha_s}{\pi} \frac{a}{\epsilon} + \left(\frac{\alpha_s}{\pi} \right)^2 \left(\frac{b}{\epsilon^2} + \frac{c}{\epsilon} \right) \tag{66}$$

determine β_1 *as a function of a*

Ejercicio 21:
Show that there is a relation between b and a. Determine β_2

Ejercicio 22:
Derive eq (159)

Ejercicio 23:
We leave as an excercise to derive the rest of the proof. Hints:(a) write the time-ordered product in term of step functions. (b)do the same for Π^* *and perform some shifts in the integration variables and use that* $\Pi_{\mu\nu}$ *is symmetric in* $\mu\nu$*. (c) use translation operators and recall the physical states have positive energy*

Ejercicio 24:
Compute $\sigma(e^+ e^- \to \mu^+ \mu^+)$*directly from dispersion relations. The computation for* $m_\mu = 0$ *is much easier, but you are encouraged to try it for* $m_\mu \neq 0$

Ejercicio 24 bis:
Could you justify on some physical arguments these limiting values?

Ejercicio 25:
Prove eq.(240)

Ejercicio 26:
Show that if we define $\xi = \frac{N}{\pi \beta_1} log \frac{t}{t_0}$ *and* $Y = log \frac{1}{x}$*, the saddle point corresponds to* $j_0 = 1 + \sqrt{\frac{\xi}{Y}}$ *and prove (254).*

Ejercicio 27:
Determine the transformation of scalar and pseudoscalar bilinears under C,P

Ejercicio 28:
Chaeck the hermiticity requirements of λ and η

Llamaremos † hermitico , * complejo conjugado

$$(\lambda\phi\bar{\Psi}\Psi)^\dagger = \lambda^*(\phi\bar{\Psi}\Psi)^\dagger \tag{67}$$

como $\bar{\Psi} = \Psi^\dagger\gamma^0$ y el campo es escalar

$$(\lambda\phi\bar{\Psi}\Psi)^\dagger = \lambda^*\phi\Psi^\dagger(\Psi^\dagger\gamma^0)^\dagger = \lambda^*\phi\Psi^\dagger\gamma^0\Psi = \lambda^*\phi\bar{\Psi}\Psi \tag{68}$$

Entonces $\lambda = lambda^*$ es real puro. Por otro lado si χ es un escalar

$$(\eta\chi\bar{\Psi}\gamma^5\Psi)^\dagger = \eta^*\chi(\bar{\Psi}\gamma^5\Psi)^\dagger \tag{69}$$

$$(\eta\chi\bar{\Psi}\gamma^5\Psi)^\dagger = \eta^*\chi\Psi^\dagger\gamma^5(\Psi^\dagger\gamma^0)^\dagger = \eta^*\chi\Psi^\dagger\gamma^5\gamma^0\Psi = -\eta^*\chi\bar{\Psi}\gamma^5\Psi \tag{70}$$

El ultimo paso se debe a que $\gamma^0, \gamma^5 = 0$. Por lo tanto $\eta = -\eta^*$ pero lo que debiamos analizar era $i\eta$ y eso implica que η sea imaginario puro.

Ejercicio 29:
Derive the above values for v and a

Ejercicio 30:
We leave as an exercise to the reader to prove that under $SU(2)_L$ transformations, $\tilde{\Phi}$ transforms in the same way as Φ

Ejercicio 31:
Check it!

Ejercicio 32:
Forget about gauge symmetries and assume that $U = U_0 \neq I$. Show that the symmetry that is left is still $SU(2)_V$. Suggestion:write $U_0 = \xi^2$, then $\xi' = L\xi h^\dagger = h\xi R^\dagger$

Ejercicio 33:
Considering the limit of small momentum transfer, find the relation between the gauge coupling constant, M_W and the Fermi constant G_F

Si comparamos el hamiltoniano de interaccion de la teoria de Fermi

$$H_{int} = \frac{G_F}{\sqrt{2}}\int d^3x J^\dagger{}_\alpha J^\alpha \tag{71}$$

que po lo encontrado en [75] podemos identificar en el acoplo de la corriente debil cargada

$$\frac{g^2}{8} = \bar{g}^2 \tag{72}$$

que es identico a

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8{M_W}^2} \tag{73}$$

pero como [77] ${M_W}^2 = \frac{v^2 g^2}{4}$ entonces podemos pensar que esta relacion es el comportamiento asintotico del propagador bosonico

$$\frac{(-i\bar{g})^2}{\sqrt{q^2 - {M_W}^2}} \overrightarrow{q^2 \to 0} \frac{g^2}{8{M_W}^2} \tag{74}$$

equivalente a bajas energias o small momentum transfer.