Compact lattice $U(1)$ and Seiberg-Witten duality

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Abstract

Simulations in compact U(1) lattice gauge theory in 4D show now beyond any reasonable doubts that the phase transition separating the Coulomb from the confined phase is of first order, albeit a very weak one. This settles the issue from the numerical side. On the analytical side, it was suggested some time ago, based on the qualitative analogy between the phase diagram of such a model and the one of scalar QED obtained by soft breaking the $N = 2$ Seiberg-Witten model down to $N = 0$, that the phase transition should be of second order. In this work we take a fresh look at this issue and show that a proper implementation of the Seiberg-Witten model below the supersymmetry breaking scale requires considering some new radiative corrections. Through the Coleman-Weinberg mechanism this turns the second order transition into a weakly first order one, in agreement with the numerical results. We comment on several other aspects of this continuum model.

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1 Introduction

$U(1)$ pure gauge theory in four dimensions is an interacting theory when formulated on the lattice. In fact there are two physically distinct phases: one characterized by massless photons and long-range interactions (the regime we would associate to continuum QED) and a strongly interacting region, where electrical charges are confined and monopoles (that have a finite energy thanks to the finite value of the lattice spacing) proliferate. It is *a priori* unclear which "continuum" theory is adequate to describe this regime.

A long-standing controversy regarding the order of the phase transition separating both regimes existed for some time. Traditionally the phase transition was believed to be of first order [1], but this belief was questioned in a series of papers [2] where some evidence was provided to suggest a second order transition. These authors believed that the apparent first order nature of the phase transition was due to some specific configurations that on a toroidal topology would lead to long-lived metastable states and these could be mistaken as footprints of a first order transition. To avoid metastability the authors performed a simulation using spherical topology. The same authors also performed a number of interesting measurements regarding the spectrum of the theory [3].

Soon afterwards, it was realized that, when going to larger lattices, a signal for a first order transition was seen also on spherical topologies [4], thus concluding that the transition was first order even if a very weak one. This seems to be the commonly accepted lore [5].

Clearly in all the above discussion an analytical approach is lacking. It would be highly desirable to gain some understanding of the mechanism of monopole condensation in this specific problem, but until very recently it was unclear how to treat these topological excitations. Furthermore, the fact that the transition was believed to be of first order would suggest that indeed there was no continuum theory associated to it. The suggestion that the transition could be of second order, combined with the results on $N = 2$ and $N = 1$ gauge theories obtained by Seiberg and Witten [6] led a group of authors to propose a continuum model [7] for it and, in fact, for the strongly coupled regime of compact lattice $U(1)$.

We shortly summarize the scenario and results presented in [7]. We shall refer in what follows exclusively to the continuum model. The starting point is $N = 2$ $SU(2)$ Yang-Mills theory, whose action is

$$S_{bare} = \int d^4x \left(\frac{1}{32\pi} Im \left(\tau \int d^2\theta Tr W^\alpha W_\alpha \right) + \int d^2\theta d^2\bar{\theta} Tr \Phi^\dagger e^{2gV} \Phi \right). \qquad (1)$$

Its scalar potential has a flat direction and in order to define the theory a complex parameter, the vacuum expectation value of the scalar field, has to be fixed, $\langle\phi\rangle = \frac{1}{2}a\tau^3$.

At scales $\mu^2 \ll u \equiv \frac{1}{2}a^2$ the charged fields decouple and the model is effectively described by a $N=2$, $U(1)$ gauge theory

$$S_{SW}^{N=2} = \frac{1}{4\pi} Im \left[\int d^4x \left(\int d^2\theta d^2\bar{\theta} \frac{\partial F}{\partial A} A^\dagger + \frac{1}{2} \int d^2\theta \frac{\partial^2 F}{\partial A^2} W^\alpha W_\alpha \right) \right] + \mathcal{O}(\frac{p^2}{\Lambda^2}), \qquad (2)$$

where F is a known function of A (the chiral superfield in the $N=2$ multiplet of the photon) and the dynamically generated scale Λ, W^α is the Abelian field strength (for details and notation we refer to [6, 8]). For scales $\mu^2 \geq u$ the charged fields are not decoupled and they contribute to the running. At $\mu^2 \simeq u$, when the charged fields decouple, the effective coupling constant, which is related to the function F by $4\pi/g^2 = Im\frac{\partial^2 F}{\partial a^2}$, freezes at the value $g = g(u)$ since below $\mu^2 = u$ no fields can contribute to its running. If $u \gg \Lambda$, this value can be safely computed within perturbation theory by using the beta function of $N=2$, $SU(2)$ Yang-Mills theory[9]. The value $g = g(u)$ is thus the appropriate one to use in the effective potential at $p^2 \to 0$.

As u decreases and approaches Λ, perturbation theory becomes unreliable. The appropriate variables to describe the theory are not g and a, but the dual [10] variables g_D and a_D, defined through the relations

$$a_D = \frac{\partial F}{\partial a} \qquad \frac{4\pi}{g_D^2} = Im\frac{\partial^2 F}{\partial a_D^2} = b_{11} \qquad (3)$$

Holomorphy and monodromy dictate that g_D runs around $g_D \sim 0$ in a way that shows that a massless hypermultiplet is present in the spectrum: it corresponds to a magnetic monopole. The appropriate value of the coupling constant for long-distance physics ($p^2 \to 0$) in this regime is $g_D(a_D)$. This is exactly for the same reasons that were described above phrased this time in term of dual variables.

The appropriate effective theory description for small values of a_D is thus provided by [8] $S = S_D^{N=2} + S_M$, where

$$\begin{aligned} S_D^{N=2} &= S_{SW}(A \to A_D, V \to V_D), \\ S_M &= \int d^4x \int d^4\theta \left(M^* e^{2V_D} M + \tilde{M}^* e^{-2V_D} \tilde{M} \right) + \int d^2\theta \sqrt{2} A_D M \tilde{M} + h.c. \end{aligned} \qquad (4)$$

and where M and $\tilde{M}$ describe the $N=2$ monopole hypermultiplet. Corrections to the above effective action will be of order p^2/Λ^2.

So far this seems to have little to do with the compact lattice $U(1)$ theory, except for the fact that the theory has a manifest $U(1)$ symmetry and that massless monopoles appear in a particular point of the moduli space of the theory. Clearly we need two more ingredients: we need to enlarge the region where light monopoles appear in order to get a finite density of monopoles $\langle m \rangle \neq 0$ and, eventually, to break $N=2$ down to $N=0$.

In [7] supersymmetry is broken in two steps. First one adds a coupling between the chiral multiplet Φ (a member of the $N=2$ hypermultiplet) and a $N=1$ chiral superfield z; as a consequence the model has now residual $N = 1$ only. The part of the action containing the new superfield is:

$$S_z = \int d^4x \int d^2\theta d^2\bar{\theta} z^\dagger z + \left(\int d^4x \int d^2\theta l z \left(w - Tr(\Phi^2) \right) + h.c \right). \tag{5}$$

It introduces two free parameters l and w. This term has the net effect of enlarging the monopole condensation region.

The limit $l \to \infty$ used in [7] to study the vacuum structure allows to integrate the z super field out using its classical equations of motion. Unlike the original mechanism of Seiberg and Witten [6] the breaking introduced by the term S_z is a hard one[1] and introduces some non-trivial quantum corrections. The obvious main effect of adding S_z to the action is to lift the degeneracy of the vacuum so $u = \frac{1}{2}a^2$ is no longer a free parameter; in fact when taking the limit $l \to \infty$ it will be kept close to w. Again, if u (now w) is large, perturbation theory is valid and a semiclassical calculation makes sense. The coupling constant now will run with the perturbative beta functions of $N = 1$ (extended with the new matter multiplet z in the case of $l \to 0$). However at some point, exactly as for $N = 2$, the perturbative procedure will break down.

Then to go from $N = 1$ to $N = 0$ and make contact with the real world, the technique discussed in [8] was used. This consists in coupling to the original $N = 2$ superfield a further $N = 2$ superfield (spurion). The action becomes

$$S_{SW}^{N=0} = \frac{1}{4\pi} Im \left[\int d^4x \int d^2\theta d^2\bar{\theta} \frac{\partial F}{\partial A^i} \bar{A}^i + \frac{1}{2} \int d^4x \int d^2\theta \frac{\partial^2 F}{\partial A_i \partial A_j} W^{i\alpha} W_\alpha^j \right], \qquad i, j = 0, 1. \tag{6}$$

The physical fields are understood to be those labeled '1'. Breaking to $N = 0$ can be achieved by giving non-zero values either to the D or F terms of the spurion (labeled '0'). In [7], the choice of the auxiliary fields to break to $N = 0$ is $D_0 \neq 0, F_0 = 0$ for the reasons mentioned there. The starting point is thus finally $S_z + S_{SW}^{N=0}$

We are now in a position to make contact with compact lattice $U(1)$. Let us first summarize the parameters that we have at our disposal. If we forget about the parameter l (we simply assume that it is large enough to enforce the constraint $u \sim w$), we have two dimensional parameters, namely w and D_0 and a dimensionless one, g, that can be traded by Λ. If we are able to place ourselves in a region where the characteristic momenta is much below both w and D_0 it is clear that we will be dealing with an effective $N = 0$, $U(1)$ gauge theory. We still have some freedom in adjusting the relative values of w and D_0; if by doing so we are able to trigger monopole condensation we shall be describing the continuum version of the confinement-Coulomb phase transition seen in compact lattice $U(1)$. If, in addition,

[1] It does not appear possible to enlarge the monopole condensation region with a soft breaking.

we manage to do all the previous manipulations in a controlled manner some quantitative predictions can be made. It is our purpose to convince the reader that this is the case.

2 The necessity of new quantum corrections

When constructing supersymmetric effective actions, the non-renormalization theorems[11] determine which perturbative and non perturbative quantum corrections are possible. The class of possible counter terms is greatly restricted when supersymmetry is broken by soft terms, like in the original proposal to go from $N = 2$ to $N = 1$ of Seiberg and Witten[6] .

In [8] $N = 2$ is broken down to $N = 0$ by the spurion mechanism allowing for a determination of the effective potential of the softly broken theory. (In fact, the first term of an expansion in powers of $\frac{F_0^2}{\Lambda^2}$ around the unbroken solution is found.[2])

The scenario studied in [7] is somewhat different. There the $N = 2$ supersymmetry of the Seiberg-Witten model is broken down to $N = 0$ by a combination of the spurion mechanism we have just discussed and a $N = 1$ hard breaking term. This one —unlike the breaking of Seiberg-Witten or the spurion mechanism— changes the monodromy of the original $N = 2$ theory non-trivially.

This issue was discussed in great detail in [7]. By considering the $U(1)_A \times U(1)_R$ charges, it is possible to see (modulo some highly plausible assumptions) that the requirement of $N = 1$ suffices to constrain the effective action in a way that is sufficient for our purposes.

Since we are concerned about the monopole condensation mechanism in compact $U(1)$ we must be in a region of moduli space close to Λ. This requires, as explained in [8], the use of the dual version of the Seiberg-Witten effective action S_{SW} completed with a term for the monopole and the dualization of S_z,

$$S = S_D^{N=0} + S_M + S_z^D, \tag{7}$$

where

$$S_D^{N=0} = S_{SW}^{N=0}(A \to A_D, V \to V_D), \tag{8}$$

$$S_z^D = \int d^4x \left(\int d^4\theta K(z, z^\dagger) \int d^2\theta l z \left\{ w - U(A_D) + \frac{\Lambda^4 z l}{w} f\left(\frac{A_D^2}{w}, \frac{\Lambda^4}{w^2} \right) \right\} + h.c. \right) \tag{9}$$

This involves an unknown function as well as an undetermined Kähler potential (originated by the quantum effects of the $N = 1$ term arising when constructing the effective action [7]) which are not calculable by symmetry considerations . We shall assume that the net effect of this term when computing the scalar potential is to adjust the value of a_D; namely we exchange w by a_D as a free adjustable parameter.

[2]As usual, terms of order p^2/Λ^2 are neglected.

This, in our understanding, does not exhaust the quantum corrections we have to include. At this point we depart from the analysis done in [7]. Let us see why the analysis presented in [7] is incomplete.

We have been quite careful about the actual meaning of the coupling constants appearing in the effective action. They are constants renormalized at the scale a_D, when the dual variables are used. This is fine as long as the coupling constants and parameters in the effective Lagrangian do not run from the scale a_D to $p^2 = 0$ where the effective potential is defined. This is the case in the $N = 2$ theory, or even if the theory is broken to $N = 1$ or $N = 0$ when the breaking occurs at a scale well below a_D. In the present case supersymmetry is severely broken (i.e. $p^2 \ll D_0 \sim a_D$) and the running needs to be taken into account. This makes the effective potential subject to radiative corrections.

We argue, in fact, that starting from energies around D_0 new quantum effects (the standard ones for non supersymmetric QFT) have to be included. To be more precise, the hierarchy of scales we have is

$$0 \leqslant p^2 \ll b_{01} D_0 \sim a_D \ll \Lambda. \tag{10}$$

Starting from the left, the first inequality is dictated by the interest in studying the non supersymmetric regime, whereas the second allows us to use the results of [8]. In fact we will use the analytical solution of Seiberg-Witten and its generalization to $N = 0$ in (7) as the Landau-Ginzburg (tree level) approximation to the complete effective action . From there we still need to scale down to zero momenta to read out the effective potential. By doing so we shall show how the agreement between the conjecture in [7] and the lattice results is recovered. In fact, the Coleman Weinberg mechanism takes place: radiative corrections transform a second order phase transition into a first order one [12] .

3 The classical vacuum structure and mass spectrum

After elimination of the auxiliary fields and the z superfield (through the limit $l \to \infty$) we are left with a scalar potential that, up to constant terms, reads

$$V = \frac{1}{2b_{11}}\left(\tilde{m}^\dagger \tilde{m} - m^\dagger m\right)^2 + \frac{2}{b_{11}}|a_D|^2\left(m m^\dagger + \tilde{m}\tilde{m}^\dagger\right) + \frac{b_{01}}{b_{11}} D_0 \left(m m^\dagger - \tilde{m}^\dagger \tilde{m}\right) \tag{11}$$

In this expression b_{01} and b_{00} have to be understood as functions of the point of the moduli (their analytical expressions can be found in [8]) and b_{11} is the gauge coupling (derived in [6]). The different supersymmetry breaking terms do not alter the dependence of b_{ij} on a_D as shown in [7] above the scale D_0. As discussed in the previous section, we take the effective potential (11) as defined at the scale a_D. In (11) the quantities a_D and D_0 are free parameters.

After rotating the fields using the $U(1) \otimes U(1)$ rigid symmetry of the above potential, it depends only on two real fields m and $\tilde{m}$. The potential can have, depending on the values of the parameters, three different minima [7]: $m = \tilde{m} = 0$ when $-2\left|a_D\right|^2 < b_{01}D_0 < 2\left|a_D\right|^2$, or $m = 0$ and $\tilde{m}^2 = -\left(2b_{11}\left(z_1^2 + z_2^2\right) - b_{01}D_0\right)$ when $b_{01}D_0 > 2\left|a_D\right|^2$, or $\tilde{m} = 0$ and $m^2 = -\left(2\left|a_D\right|^2 + b_{01}D_0\right)$ when $b_{01}D_0 < -2\left|a_D\right|^2$.

Here we will focus on the last case (The second and the third one are interchangeable, depending on the sign of D_0). In order to study the critical region we introduce as control parameter the combination

$$\alpha = -\left(2\left|a_D\right|^2 + b_{01}D_0\right). \tag{12}$$

In fact, the mass of the monopole field (responsible for the critical behavior) is given by

$$M_m^2 = -\frac{1}{b_{11}}\left(2\left|a_D\right|^2 + b_{01}D_0\right) \tag{13}$$

and is thus proportional to α. We demand (to stay close to the transition) $\alpha \to 0$, while both D_0 and a_D are understood to be large compared to the physical scales (to decouple the supersymmetric modes), but small compared to Λ (in order to trust the Seiberg-Witten effective action).

In the region we are considering the only light scalar field is the monopole. The $\tilde{m}$ field has a large mass

$$M_{\tilde{m}}^2 = \frac{8\left|a_D\right|^2}{b_{11}}, \tag{14}$$

and it can be safely eliminated from the effective potential which is just

$$V = \frac{1}{2b_{11}}\left(m_1^2 + m_2^2\right)^2 + \frac{\alpha}{b_{11}}\left(m_2^2 + m_1^2\right) \tag{15}$$

where we have restored the imaginary part of the monopole field m_2. This is of course an ordinary $\lambda\phi^4$ potential.

As for the fermions, we have the following non diagonal mass matrix

$$\frac{1}{2b_{11}}\begin{pmatrix} 0 & a_D & 0 & i\alpha \\ a_D & 0 & \alpha & 0 \\ 0 & \alpha & 0 & \frac{1}{16\pi}\left(\frac{\partial^3\mathcal{F}}{\partial^2 a_1 \partial a_0}\right)D_0 \\ -i\alpha & 0 & \frac{1}{16\pi}\left(\frac{\partial^3\mathcal{F}}{\partial^2 a_1 \partial a_0}\right)D_0 & 0 \end{pmatrix} \tag{16}$$

where the rows and columns are ordered as $\psi_m, \psi_{\tilde{m}}, \psi_1, \lambda_1$ (see [8] for notation). This matrix can be diagonalized to obtain two physical fermions with masses close to a_D and two with masses closed to $\frac{1}{16\pi}\left(\frac{\partial^3\mathcal{F}}{\partial^2 a_1 \partial a_0}\right)D_0$. Both groups are extremely heavy and can be dropped from the effective action using their equation of motion.

4 One-Loop effective potential and phase diagram

Using the results of Seiberg and Witten as 'boundary conditions' at the scale $a_D \sim b_{01} D_0$ we are now in a position to run the effective potential down from $\mu = a_D$ to $\mu = 0$. The only light fields that are left are as expected the dual photon and the monopole field. This is however not a generic potential: some peculiar relations between self and electric couplings are inherited from supersymmetry. But of course these relations shall not be preserved as supersymmetry is broken below $D_0 \sim a_D$.

The full Lagrangian, counter terms included, will be

$$\begin{aligned}
\mathcal{L} \;=\; & \left(\partial^\mu m_1 + \frac{1}{\sqrt{b_{11}}} m_2 A^\mu\right)^2 + \left(\partial_\mu m_2 - \frac{1}{\sqrt{b_{11}}} A_\mu m_1\right)^2 + \frac{1}{2}\left(\left(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu\right) A_\mu A_\nu\right) \\
& - \frac{1}{2b_{11}}\left(m_1^2 + m_2^2\right)^2 - \frac{\alpha}{b_{11}}\left(m_2^2 + m_1^2\right) \\
& + \frac{\delta b_{11}}{2b_{11}^2}\left(m_1^2 + m_2^2\right)^2 + 2\delta b_e \left(\partial_\mu m_1 A^\mu m_2\right) - 2\delta b_e \left(\partial_\mu m_2 A^\mu m_1\right) \\
& + \frac{2\delta b_e}{\sqrt{b_{11}}}\left(m_2^2 + m_1^2\right) A^\mu A_\mu - \frac{\delta M^2}{b_{11}}\left(m_2^2 + m_1^2\right).
\end{aligned} \tag{17}$$

We shall work in the Landau gauge.

The one loop correction to the effective potential in $\overline{MS}$ produces the following effective potential

$$\begin{aligned}
V \;=\; & \frac{1}{2b_{11}} m^4 + \frac{\alpha}{b_{11}} m^2 + \frac{1}{(4\pi)^2}\left(\frac{3}{b_{11}} m^2 + \frac{\alpha}{b_{11}}\right)^2 \left(\log \frac{3m^2 + \alpha}{\mu^2} - \frac{3}{2}\right) \\
& + \frac{1}{(4\pi)^2}\left(\frac{1}{b_{11}} m^2 + \frac{\alpha}{b_{11}}\right)^2 \left(\log \frac{m^2 + \alpha}{\mu^2} - \frac{3}{2}\right) + \frac{3}{(4\pi)^2} \frac{m^4}{b_{11}^2}\left(\log \frac{m^2}{\mu^2} - \frac{5}{6}\right)
\end{aligned} \tag{18}$$

in agreement with [13], for instance. We have used again the residual $U(1)$ rigid symmetry to get rid of m_2 and we have replaced $m_1 \to m$. Here μ^2 is the subtraction scale.

From the one loop calculations we also extract the beta functions

$$\beta_{b_{11}} = \frac{1}{(4\pi)^2} \frac{13}{b_{11}^2} \qquad \beta_{b_e} = \frac{1}{24\pi^2}\left(\frac{2}{b_{11}}\right)^{\frac{3}{2}}, \tag{19}$$

which are of course different.

With these results what follows is quite predictable. We are interested in the phase transition from the Coulomb to the confined phase. The order parameter is m. A value of m different from zero reveals monopole condensation and the presence of the dual Meissner effect.

By changing α we move from a phase with $m \neq 0$ to one with $m = 0$. If we tune α to have zero renormalized mass (to stay on the critical surface) the presence of a minimum in the potential outside from the origin implies a first order phase transition [12].

On the critical surface, there are in the effective theory two distinct coupling constants (the gauge coupling b_e and the quartic self coupling for the scalar field b_{11}) that have the same bare value , as dictated by supersymmetry. We call this bare value b_{11}^{SW}. In spite of this, they are different below a_D as they run with different beta functions (a naive inspection of the bare potential would lead to the erroneous conclusion that there is just one coupling). The existence of two beta functions, as pointed out by Yamagishi [14], is the essential ingredient that allows, in scalar QED, (as in other models [15]) radiative corrections to induce a first order transition [12]. Following the analysis in [14] we obtain the one loop RG improved effective potential

$$V = \frac{b_{11}(t)}{4!} m^4 exp\left[4\int_0^t dt' \frac{\gamma(t')}{1-\gamma(t')}\right] \tag{20}$$

where $t = ln(m/\mu)$ and $\gamma(t) = \gamma(b_{11}(t), b_e(t))$ with $b_e(t), b_{11}(t)$ solution of the differential equations

$$\frac{db_{11}(t)}{dt} = \frac{\beta_{b_{11}}(b_{11}, b_e)}{1-\gamma(b_{11}, b_e)}; \qquad \frac{db_e(t)}{dt} = \frac{\beta_{b_e}(b_{11}, b_e)}{1-\gamma(b_{11}, b_e)} \tag{21}$$

with initial conditions

$$b_{11}(0) = b_{11}^{SW} \qquad b_e(0) = b_{11}^{SW} \tag{22}$$

The existence of a minimum away from the origin implies that the trajectories (21) in the (b_{11}, b_e) plane should cross the line

$$4b_{11} + \beta_{b_{11}} = 0. \tag{23}$$

The plot of the renormalization-group trajectories of the coupling constants (tangent to the arrow field in figure 1) shows that the trajectories leaving the bare curve $\frac{1}{b_{11}} = \frac{b_e^2}{2}$ (upper solid line in figure 1) always cross the line (23) (lower solid line in figure 1). This implies that the transition is of first order in all the plane but the origin. A zoom of the previous plot in the perturbative region with explicitly sketched flow is drawn in figure 2.

The fact that the transition is first order can also be checked by direct inspection of the potential (18); that is by plotting (18) for typical values of the parameters and looking to the position of the minima.

It is evident that the system passes from the Coulomb phase to the confined one through a first order transition (fig. 4). We thus conclude that the transition is indeed of first order. However, as befits a radiatively induced first order transition, this is expected to be quite weak, exactly as observed in lattice simulations.

5 Conclusions

The agreement between the predictions of Seiberg-Witten theory and what is observed in lattice simulations of compact $U(1)$ goes beyond the fact that both indicate the existence of a weakly first order transition. As it was emphasized in [7], the parity and charge-conjugation quantum numbers of the lightest one-particle states in the confinement phase (0^{++} and 1^{+-}) seem to agree with the clearest signal of "gauge-ball" state from lattice simulation [3]. The fact that the transition is weekly first order makes the comparison between continuum and lattice results meaningful.

It might seem that, at the end of the day after the successive supersymmetry breakings, very little is left of the bare Lagrangian with $N = 2$, $SU(2)$ Yang-Mills gauge invariance and that one is left with the Lagrangian of scalar electrodynamics. This perception is indeed correct, but it was by no means obvious that this was the appropriate theoretical framework to describe the confining-Coulomb transition in compact lattice $U(1)$. It was not even obvious that the appropriate theory could be written in terms of local variables. The good theoretical control provided by the Seiberg-Witten model allows us to be on firm ground.

The supersymmetry breaking is well under control. The problem we are interested is considerably easier than trying to get predictions for, say, $SU(2)$ gauge theory, because we are in a kinematical regime where it is consistent to have $D_0 \ll \Lambda$. As far as we can see, the theoretical prediction is quite robust.

The results obtained here modify the conclusions of [7]. Some radiative corrections, necessary below the supersymmetry breaking scale, were overlooked in that work. Their consideration reconciles the qualitative agreement between the phase diagram of the $U(1)$ theory obtained after breaking the supersymmetry of the Seiberg-Witten theory and the phase diagram of compact $U(1)$ lattice theory in four dimensions. This also supports the idea that confinement in four dimensions is due to monopole condensation as already demonstrated in the three-dimensional case [16]. We are currently considering the application of the same techniques to other models.

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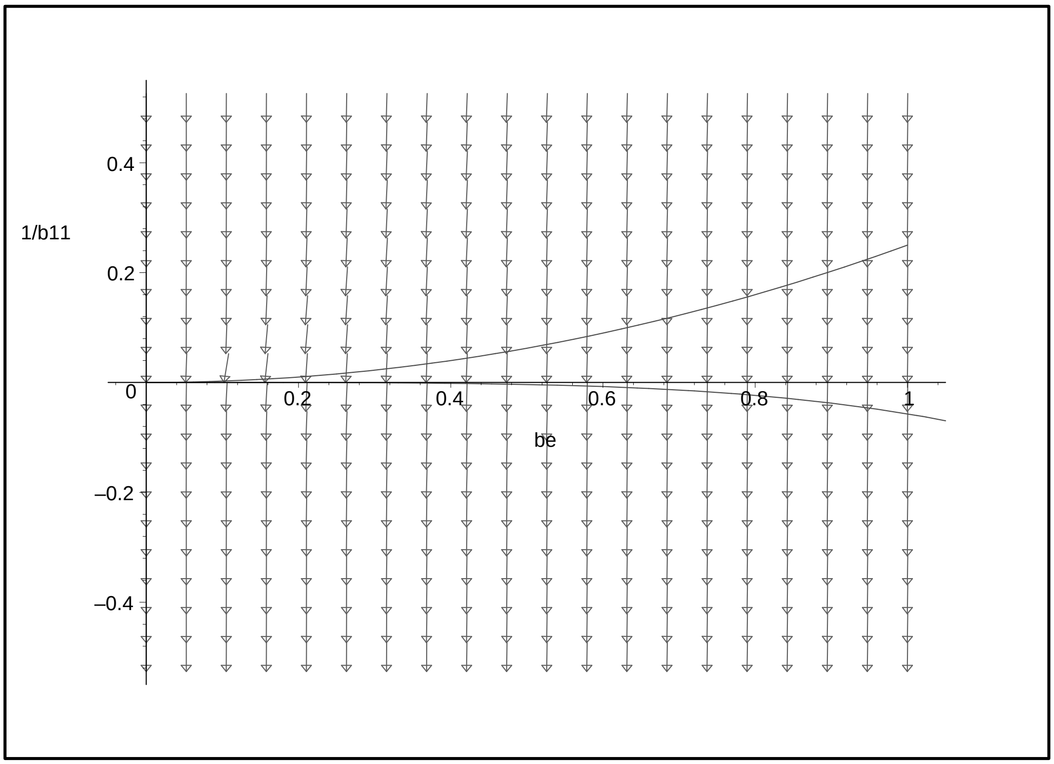

Figure 1: Flux of the couplings along the renormalization group trajectories, b_{11} versus b_e. Starting with bare conditions on the upper line the flux always crosses the lower line representing 23. This is the footprint of the first order transition

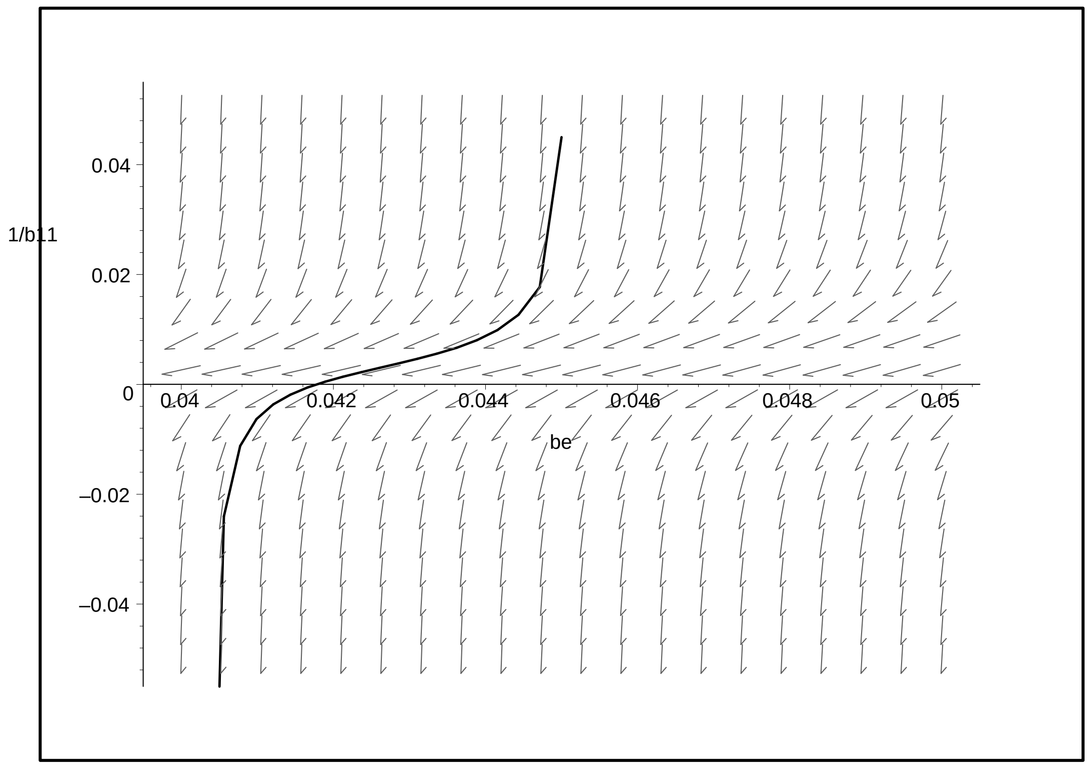

Figure 2: Zoom of the previous plot in the perturbative region (small b_e). The continuous line sketches the flux of the couplings along the renormalization group trajectories in the b_{11},b_e plane for some typical initial conditions

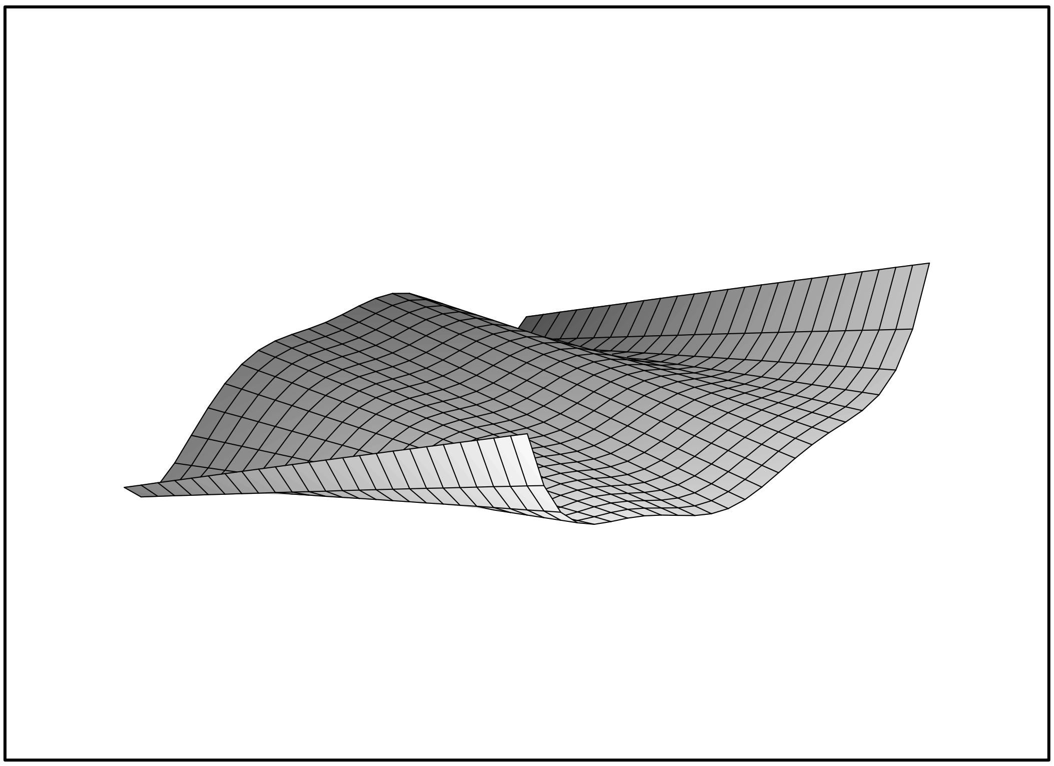

Figure 3: Dynamical evolution: from Coulomb to confined through a first order transition. Potential as function of m and α.

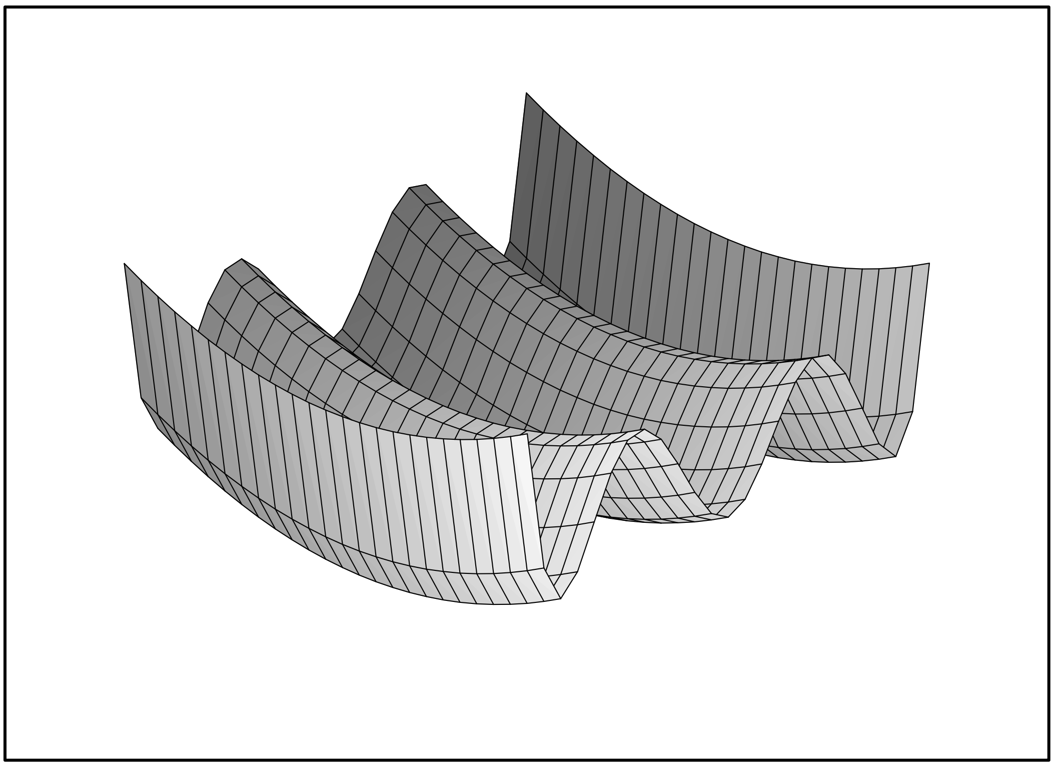

Figure 4: First order phase transition: the potential as function of m and $\tilde{m}$ with $\alpha = 0.008$ shows the coexistence of the Coulomb and confined phases.

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