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Vector meson decays from the Extended Chiral Quark Model

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We derive the the effective lagrangian that describes the interactions among vector, axial-vector mesons and pseudoscalars starting from the extended chiral quark model (ECQM). The results for the low-energy constants of this effective lagrangian have a parametric resemblance with existing predictions based on the Nambu-Jona-Lasinio model (except for some overall signs that we correct), but are numerically different. Therefore a precise measurement of these decay constants can shed some light on the way chiral symmetry breaking is modelled in QCD. Although most of the constants are poorly measured, comparison with phenomenology allows us to determine one of the parameters of the ECQM that could not be fully determined in previous analyses.

INTRODUCTION

The chiral quark model (CQM) [1, 2, 3] gives a good phenomenogical description of chiral symmetry breaking and reasonable values for the Gasser Leutwyler cofficients, but does not describe meson states with masses ~ 1 GeV and does not provide a model for chiral symmetry breaking; it simply assumes that this takes place and incorporates the lowest dimensional operators compatible with the symmetry breaking pattern. The Nambu Jona Lasinio Model (NJL) [4, 5, 6, 7] does provide a specific model for chiral symmetry breaking by assuming strong attractive forces in the scalar channel. It predicts a light narrow scalar partice, the elusive σ particle, the would-be chiral partner of the pion. But unitarization studies combined with the large N_c limit[8] suggest that such a particle is a dynamical resonance and not a truly QCD narrow resonance. Thus this simple model of chiral symmetry breaking is clearly disfavoured.

The possibility that the phenomenology of low energy QCD can be captured by an hybrid model, where some features of both models are retained, was investigated in [9, 10]. The aim in these works was to write a very general low-energy model of QCD containing all possible operators compatible with the symmetries of the model and then let phenomenology decide the respective importance of the different terms. The model is understood to be valid in the chirally broken phase (so like in the CQM, no specific model of chiral symmetry breaking is assumed). In this model the pion stands alone, and the partner of the σ particle (that is identified with a well established resonance, the $f_0(980)$ in the isoscalar channel) is the $\pi'(1300)$ in the isovector channel. The authors named this model Extended Chiral Quark Model (ECQM).

In this work we shall explore some of the phenomenological consequences of the ECQM in the realm of vector and axial-vector decays. We shall argue later what is the phenomenological interest of understanding these decays. For us they are basically a testing ground of the ECQM. It will be of interest to us also to compare the predictions of the ECQM to those of the NJL model. As we shall see the comparison is interesting to understand the criticality of the models on various parameters.

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We shall first review the extended chiral quark model of [9, 10]. After we will present our derivation of the effective lagrangian for vector and axial-vector mesons, then present our numerical predictions for the low-energy constants and conclusions.

THE EFFECTIVE CHIRAL QUARK MODEL

The extended chiral quark model, ECQM, was introduced in [9, 10]. The reader is referred to these works for further details as we here present a succint description only. In Euclidean conventions its lagrangian consists of three different terms

$$\mathcal{L}_{ECQM} = \mathcal{L}_{ch} + \mathcal{L}_{\mathcal{M}} + \mathcal{L}_{vec}, \tag{1}$$

where

$$\begin{aligned}
\mathcal{L}_{ch} \;=\;& \mathcal{L}_0 + i\bar{Q}\left(\not{D} + M_0\right)Q + i\frac{4\delta f_0}{\Lambda^2}\bar{Q}a_\mu a_\mu Q \\
&+\frac{G_{S0}}{4N_c\Lambda^2}\left(\bar{Q}_L Q_R + \bar{Q}_R Q_L\right)^2 - \frac{G_{P1}}{4N_c\Lambda^2}(-\bar{Q}_L\vec{\tau}Q_R + \bar{Q}_R\vec{\tau}Q_L)^2 \\
&+\frac{G_{S1}}{4N_c\Lambda^2}(\bar{Q}_L\vec{\tau}Q_R + \bar{Q}_R\vec{\tau}Q_L)^2 - \frac{G_{P0}}{4N_c\Lambda^2}(-\bar{Q}_L Q_R + \bar{Q}_R Q_L)^2,
\end{aligned} \tag{2}$$

$$\begin{aligned}
\mathcal{L}_{\mathcal{M}} \;=\;& i(\frac{1}{2} + \epsilon)\left(\bar{Q}_R\mathcal{M}Q_L + \bar{Q}_L\mathcal{M}^\dagger Q_R\right) \\
&+i(\frac{1}{2} - \epsilon)\left(\bar{Q}_R\mathcal{M}^\dagger Q_L + \bar{Q}_L\mathcal{M}Q_R\right) \\
&+\langle c_0\left(\mathcal{M} + \mathcal{M}^\dagger\right) + c_5(\mathcal{M} + \mathcal{M}^\dagger)a_\mu a_\mu + c_8\left(\mathcal{M}^2 + \left(\mathcal{M}^\dagger\right)^2\right)\rangle,
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
\mathcal{L}_{vec} =\; -&\; \frac{G_{V1}}{4N_c\Lambda^2}\bar{Q}\vec{\tau}\gamma_\mu Q\bar{Q}\vec{\tau}\gamma_\mu Q - \frac{G_{A1}}{4N_c\Lambda^2}\bar{Q}\vec{\tau}\gamma_5\gamma_\mu Q\bar{Q}\vec{\tau}\gamma_5\gamma_\mu Q \\
-&\; \frac{G_{V0}}{4N_c\Lambda^2}\bar{Q}\gamma_\mu Q\bar{Q}\gamma_\mu Q - \frac{G_{A0}}{4N_c\Lambda^2}\bar{Q}\gamma_5\gamma_\mu Q\bar{Q}\gamma_5\gamma_\mu Q \\
+&\; c_{10}\langle U\bar{L}_{\mu\nu}U^\dagger\bar{R}_{\mu\nu}\rangle.
\end{aligned} \tag{4}$$

The notation we have used is the following: Q are the quark fields written in the 'constituent' or 'rotated' basis,

$$Q_L = uq_L, \qquad Q_R = u^\dagger q_R, \qquad u^2 = U = \exp(2i\pi/F_0), \qquad \mathcal{M} = u^\dagger m u^\dagger \tag{5}$$

m is the quark mass matrix, $\not{D}$ is the covariant derivative defined as

$$\not{D} \equiv \not{\partial} + \not{v} - \gamma_5\tilde{g}_A \not{a}, \tag{6}$$

with the (antihermitian) fields

$$v_\mu = \frac{1}{2}\left(u^\dagger\partial_\mu u - \partial_\mu u u^\dagger + u^\dagger\bar{V}_\mu u + u\bar{V}_\mu u^\dagger - u^\dagger\bar{A}_\mu u + u\bar{A}_\mu u^\dagger\right) \tag{7}$$

$$a_\mu = \frac{1}{2}\left(-u^\dagger\partial_\mu u - \partial_\mu u u^\dagger - u^\dagger\bar{V}_\mu u + u\bar{V}_\mu u^\dagger + u^\dagger\bar{A}_\mu u + u\bar{A}_\mu u^\dagger\right), \tag{8}$$

where $\bar{A}$ and $\bar{V}$ are external axial and vector fields.

The parameter M_0 is the so called 'constituent' mass. G_{S0}, G_{P1}, G_{V1} and G_{A1} are constants parametrizing the four-fermion interactions (indices denote the corresponding J, I channels). These couplings will eventually be reduced and fixed by comparing with the physical values of vector meson masses. Λ is a physical UV cut-off identified with the scale of chiral symmetry breaking ($\simeq 1.4$ GeV).

The reader has by now undoubtedly noticed that $\mathcal{L}_{ch}$ contains the usual term operators in the CQM plus some four-quark operators (reminiscent of the NJL). However we intend to describe physics in the chirally broken phase and our fields include the pion matrix u, as befits an effective theory that should retain only the light degrees of freedom. Also, unlike in NJL the quark degrees of freedom appearing in (1) are from the very beginning 'constituent' quarks, quarks dressed by pions below the chiral symmetry breaking scale. The scalar and pseudoscalar four-quark couplings in (2) need not be equal in order to preserve chiral symmetry (again unlike in NJL models).

An additional operator is allowed by symmetry: $\mathcal{L}_{\mathcal{M}}$ contains the dependence on current quark masses. Again, because of the possibility of including the u field, the structure of this term is quite rich. Finally, $\mathcal{L}_{vec}$ includes four quark operators in the vector and axial-vector channels.

The term

$$\mathcal{L}_0 = -\frac{f_0^2}{4}\langle a_\mu a^\mu \rangle. \tag{9}$$

as well as the operators whose coefficient are c_0, c_5, c_8 and c_{10} contain contributions from those degrees of freedom with masses $\leq \Lambda \simeq 1.4$ GeV. These (c_0, c_5, c_8 and c_{10}) contributions are typically small, the bulk of the contribution coming from the light resonances. They are unimportant for our present discussion as is δf_0 in (2).

The effective lagrangian in eq. (2) is the most general[1] one compatible with the principles of gauge and chiral invariance, CP invariance and locality that one can build out of quarks and pions up to, and including, operators of dimension six. It contains four-fermion pieces somewhat reminiscent of NJL, but the philosophy is different here: these terms typically will not have large coefficients to trigger chiral symmetry breaking. No specific mechanism is assumed for the latter, we just write an effective lagrangian that is compatible with it. The vector field $\bar{V}$ contains a piece that commutes with u describing the residual gluon interactions that ultimately ensure confinement.

Some of the constants and terms are somewhat non-standard. For instance, the naïve QCD value for the parameter ϵ is $\epsilon = 0.5$, but its actual value in the low energy theory is largely unconstrained. We shall return to this later.

After introducing auxiliary fields in all four channels, the effective lagrangian (1) becomes bilinear in the quark fields. The four-fermion interaction is replaced by

$$\bar{Q}\left[i\widetilde{\Sigma} - \gamma_5\widetilde{\Pi} + \frac{1}{2}\gamma_\mu\widetilde{V}_\mu + \frac{1}{2}\gamma_\mu\gamma_5\widetilde{A}_\mu\right]Q + 2N_c\Lambda^2\left[\frac{\widetilde{\Sigma}^2}{G_{S0}} + \frac{(\widetilde{\Pi}^a)^2}{G_{P1}} + \frac{\left(\widetilde{V}_\mu^a\right)^2}{4G_{V1}} + \frac{\left(\widetilde{A}_\mu^a\right)^2}{4G_{A1}}\right] \tag{10}$$

and we include an integration over the real auxiliary variables $\widetilde{\Sigma}, \widetilde{\Pi}^a, \widetilde{V}_\mu^a, \widetilde{A}_\mu^a$, defined by $\widetilde{\Pi} \equiv \widetilde{\Pi}^a\tau^a/\sqrt{2}$, $\widetilde{V}_\mu = \widetilde{V}_\mu^a\tau^a/\sqrt{2}$, etc. (note that the fields $\widetilde{V}_\mu^a$ and $\widetilde{A}_\mu^a$ are hermitian). This operation amounts to the replacement

$$v_\mu \to \mathcal{V}_\mu = v_\mu - \frac{1}{2}i\widetilde{V}_\mu, \qquad \tilde{g}_A a_\mu \to \mathcal{A}_\mu = \tilde{g}_A a_\mu - \frac{1}{2}i\widetilde{A}_\mu, \tag{11}$$

and to the addition of scalar (Σ) and pseudoscalar (Π) fields in the Dirac operator

$$\Sigma = M_0 + \widetilde{\Sigma} + \frac{1}{2}\left(\mathcal{M} + \mathcal{M}^\dagger\right) + \frac{4\delta f_0}{\Lambda^2}a_\mu a_\mu, \qquad \Pi = \widetilde{\Pi} + i\epsilon\left(\mathcal{M}^\dagger - \mathcal{M}\right), \tag{12}$$

which becomes

$$\hat{D} = \partial\!\!\!/ + \mathcal{V}\!\!\!/ - \gamma_5\tilde{g}_A\mathcal{A}\!\!\!/ + \Sigma + i\gamma_5\Pi \tag{13}$$

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[1] except for the fact that for simplicity not all isospin channels are included

We can now integrate out the bilinear quarks and solve for the mass gap. In the weak coupling regime the solution becomes

$$\Sigma_0 \simeq M_0 + m, \tag{14}$$

m being the current quark mass. In practice the constituent mass is large enough so that a derivative expansion in inverse powers of Σ_0 makes sense at least for some range of energies. We can thus write the full quark-loop effective action. Retaining only the logarithmically enhanced part we get [9, 10]

$$\begin{aligned}\mathcal{L}_{1-loop} \;\simeq\; & \frac{N_c}{16\pi^2}\ln\frac{\Lambda^2}{\Sigma_0^2}\,\langle(\Sigma^2+\Pi^2)^2 + (\partial_\mu\Sigma)^2 + [D_\mu^{\mathcal{V}},\Pi]^2 \\ & -4(\mathcal{A}_\mu)^2\Sigma^2 - \{\mathcal{A}_\mu,\Pi\}^2 - 4i[D_\mu^{\mathcal{V}},\Pi]\;\mathcal{A}_\mu\;\Sigma\; + 2i\partial_\mu\Sigma\{\mathcal{A}_\mu,\Pi\} \\ & -\frac{1}{6}\left((F_{\mu\nu}^L)^2 + (F_{\mu\nu}^R)^2\right)\rangle. \end{aligned} \tag{15}$$

$F_{L,R}$ are field strengths constructed with $\mathcal{V}\pm\mathcal{A}$ and $D^{\mathcal{V}}$ is the covariant derivative associated to the connection $\mathcal{V}_\mu$.

In addition, we have the mass terms for the fields $\widetilde{\Sigma}$, $\widetilde{\Pi}$, $\widetilde{V}_\mu$ and $\widetilde{A}_\mu$ coming from (10). In the axial channel there is some mixing between a_μ and $\tilde{A}_\mu$; the corresponding mass term reads

$$\frac{N_c I_0 \Sigma_0^2}{4}\langle\frac{1}{\bar{G}_A}\widetilde{A}_\mu^2 + \left(i2\tilde{g}_A a_\mu + \widetilde{A}_\mu\right)^2\rangle. \tag{16}$$

The coupling $\bar{G}_A$ is introduced so as to give a natural scale for the four-fermion terms (they turn out to be ~ 0.1)

$$\bar{G}_A = 2G_{A1}I_0\frac{\Sigma_0^2}{\Lambda^2} \qquad I_0 = \frac{1}{4\pi^2}\ln\frac{\Lambda^2}{\Sigma_0^2}. \tag{17}$$

This mass term can be diagonalized by defining

$$i2\tilde{g}_A a_\mu + \widetilde{A}_\mu = i2g_A a_\mu + \frac{1}{\lambda_-}A_\mu, \tag{18}$$

with

$$g_A = \frac{\tilde{g}_A}{1+\bar{G}_A}, \tag{19}$$

We refer to [10] for details. A_μ is, finally the physical axial-vector field. In the vector field there is no mixing. Of course we have to bear in mind that we are still in Euclidean space-time. These expressions differ from the related expression in the extended NJL model due to presence of a bare constant $\tilde{g}_A$. The constant λ_- is determined by requiring proper normalization of the kinetic term for the A_μ field. One proceeds likewise for $\widetilde{V}_\mu$ finding that the properly normalized fields is $V_\mu = \lambda_+\widetilde{V}_\mu$. Furthermore one finds that

$$\lambda_+^2 = \lambda_-^2 = \frac{N_c I_0}{6}. \tag{20}$$

The values of the physical masses of the axial and vector mesons in terms of the parameters of the model can be found in Ref. [10]. Ref. [10] concentrated on the implications of the model in two-point correlators. There it was seen that, after implementing the short distance constraints coming from QCD via the Operator Product Expansion, in spite of its relatively large number of parameters, the model could be well constrained and some clear predictions emerged, comparing very favourably with the data. All the parameters in the ECQM can be thus determined (with one exception to be mentioned below).

There are two possible values for ϵ that are compatible with the fit of the two-point correlators and their subsequent matching to the OPE. This ambiguity will be resolved in this work.

It was also seen in [9, 10] that the description of the low energy phenomenology that the extended chiral quark model provides is clearly superior to that of the NJL model.

TABLE I: The parameters of the ECQM as determined in reference [10]

Λ	1.3 GeV
Σ_0	200 MeV
g_A	0.55
ϵ	0.05 / -0.51

VECTOR AND AXIAL-VECTOR PHENOMENOLOGICAL LAGRANGIANS

In what follows we want to explore other phenomenological consequences of the extended chiral quark model by deriving the effective lagrangian relevant for the decay of vector and axial vector mesons. All predictions will be essentially parameter-free, as the model is rigidly fixed from the two-point correlators as we have just indicated. The predictions for vector meson decays at order p^3 are actually contained in the first term in the expansion of the determinant of the deneralized Dirac operator (15).

Let us introduce some notations and relations

$$\nabla_\mu X \equiv \partial_\mu X + [v_\mu , X] , \qquad X_{\mu\nu} \equiv \nabla_\mu X_\nu - \nabla_\nu X_\mu , \tag{21}$$

where $X = V, A$.

$$v^{\mu\nu} \equiv \partial^\mu v^\nu - \partial^\nu v^\mu + [v^\mu , v^\nu] \tag{22}$$

$$-\frac{i}{2} f_+^{\mu\nu} \equiv v^{\mu\nu} - \frac{1}{4}[u^\mu , u^\nu] \tag{23}$$

$$f_-^{\mu\nu} \equiv \nabla^\mu u^\nu - \nabla^\nu u^\mu \tag{24}$$

where $u_\mu = -2ia_\mu$ is introduced to conform to the standard notation.

Let us now consider the most general strong lagrangian linear in the vector field and up to $\mathcal{O}(p^3)$ assuming nonet symmetry. It reads [11, 12]

$$\begin{aligned} \mathcal{L}_V \;=\; & -\frac{f_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle - \frac{ig_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu , u^\nu] \rangle \\ & + i\alpha_V \langle V_\mu [u_\nu , f_-^{\mu\nu}] \rangle + \beta_V \langle V_\mu [u^\mu , \chi_-] \rangle \end{aligned} \tag{25}$$

In the above expresion

$$\chi_\pm = 2B_0(u^+ m u^+ \pm u m^\dagger u) \qquad B_0(1\mathrm{GeV}) \simeq 1.5 \text{ GeV} \tag{26}$$

We do not include the odd-parity part in the above lagrangian (proportional to $\epsilon^{\alpha\beta\mu\nu}$). For axial-vector fields

$$\begin{aligned} \mathcal{L}_A \;=\; & -\frac{f_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle + i\alpha_A \langle A_\mu [u_\nu , f_+^{\mu\nu}] \rangle + \gamma_1 \langle A_\mu u_\nu u^\mu u^\nu \rangle \\ & + \gamma_2 \langle A_\mu \{ u^\mu , u^\nu u_\nu \} \rangle + \gamma_3 \langle A_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + \gamma_4 \langle A_\mu u^\mu \rangle \langle u_\nu u^\nu \rangle \end{aligned} \tag{27}$$

Again the terms containing $\varepsilon_{\mu\nu\rho\sigma}$ will not be considered in this work. Note that there is some ambiguity in the choice of overall signs. We choose eventually the sign of the axial field so as to conform to the usual conventions (implying a positive f_A).

TABLE II: The predictions of the ECQM for the vector couplings compared to experiment

	ECQM	Experiment
f_V	input	0.20
g_V	0.07	0.09
α_V	0.02	-
β_V	-0.008	-0.018

In order to make contact with phenomenology, we have to Wick rotate the Euclidean effective lagrangian we have obtained. Using the previous expressions, from the extended Chiral Quark model the following predictions emerge in the vector and axial-vector meson sector

$$f_V^2 = N_c I_0/6, \qquad g_V = f_V \frac{1-g_A^2}{2}, \qquad \alpha_V = f_V \frac{g_A^2}{2\sqrt{2}}, \qquad \beta_V = f_V \frac{3 g_A M_0 \epsilon}{2\sqrt{2} B_0}, \tag{28}$$

$$f_A = f_V g_A, \qquad \alpha_A = f_V \frac{g_A}{2\sqrt{2}}, \qquad \gamma_A^1 = -f_V \frac{g_A(1-g_A^2)}{2\sqrt{2}}, \qquad \gamma_A^2 = f_V \frac{g_A(1-g_A^2)}{4\sqrt{2}}. \tag{29}$$

These are the predictions of the ECQM.

When comparing to the predictions of the NJL model [12], we note that, although the details of the expressions between our results and those of the NJL model obviously differ, when looking at the leading term in NJL there is an overall change of sign in the axial-vector couplings $(\alpha_A, \gamma_A^1, \gamma_A^2)$ and also in β_V and α_V, perhaps due to different conventions in Minkowski and Euclidean space. An overall change of sign everywhere is of course undetectable.

NUMERICAL ANALYSIS AND CONCLUSIONS

The previous predictions, making use of the 'best fit' presented in Table I lead to the set of numerical values quoted in Tables II and III.

While there is no need to stress here the relevance of g_V , f_V and f_A it is worth emphasizing the phenomenological impact of the other couplings. Vector meson dominace in weak non-leptonic kaon decays have been studied accurately [13, 14]. In fact it has been shown in Ref. [14] that there are several cases where the remaining couplings are particularly interesting due the vanishing of the contributions due to g_V and f_V are: see $K \to 2\pi/3\pi$ and $K \to \pi^+\pi^0\gamma$ [14]. The couplings in $\mathcal{L}_V$ and $\mathcal{L}_A$ can be determined, in principle, from the phenomenology of the vector meson decays. $|f_V|$ and $|\alpha_V|$ could be obtained from the experimental widths [15] of $\rho^0 \to e^+e^-$, $\omega \to \pi^0\gamma$, $\omega \to \pi\pi\pi$ and $\rho \to \pi\pi\gamma$, respectively, while g_V and β_V enter in $\rho \to \pi\pi$.

As for the axial-vector couplings they can be determined from the decays $a_1^+ \to 3\pi$ (γ_1, γ_2, γ_3 and γ_4) and $a_1^+ \to \pi^+\gamma$ (f_A and α_A)[12]. However unfortunately data are not precise enough to go beyond a good determination of f_A.

In Table 2 we collect the experimental determinations (when available). As we have emphasized the predictions are absolutely rigid, as all the free parameters in the model are fixed beforehand from the two-point functions.

When comparing the experimental value for β_V with the theoretical prediction of the model, this favours the value $\epsilon = -0.5$. That solves the ambiguity in the determination of ϵ we alluded to before and fixes completely the leading coupling constants of the ECQM.

It is unfortunate that except for g_V and β_V the other couplings in this parity even sector are not measured yet. In some of them we get results that clearly differ numerically from the predictions of the

TABLE III: The predictions of the ECQM for the axial-vector couplings

	ECQM	Experiment
f_A	0.11	0.097
α_A	0.04	-
γ_1	-0.03	-
γ_2	0.01	-
$\gamma_{3,4}$	$\mathcal{O}(1/\sqrt{N_c})$	-

NJL model and therefore they provide a clear test of the mechanisms of chiral symmetry breaking. Their measurement is clearly interesting.

Even if we have reduced ourself to the study of non-anomalous vector and axial coupling some interesting conclusions on NJL and ECQM can be drawn. Our numerical values certainly differ from the NJL ones and thus measuring the low energy constants related to meson decays into pseudoscalars can be particularly telling about the mechanisms of chiral symmetry breaking in QCD and its modelization. We have been able to resolve the ambiguity in the determination of the ϵ parameter in the ECQM.

Particularly useful are some VMD couplings which could be measured in the near future and might be phenomenological relevant in K-meson decays.

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